saylor.org ACADEMY

Saylor Academy awards

Dan Rimniceanu

this certificate of achievement for

MA005: Calculus I

31 mai 2018
Issue Date

11602641
Certificate ID

saylor.org ACADEMY

Saylor Academy awards

Dan Rimniceanu

this certificate of achievement for

MA101: Single-Variable Calculus

7 mai 2018
Issue Date

11526666
Certificate ID

Calculus can be thought of as the mathematics of *change*. Because everything in the world is changing, calculus helps us track those changes. Algebra, by contrast, can be thought of as dealing with a large set of numbers that are inherently *constant*. Solving an algebra problem, like $y=2x+5$, merely produces a pairing of two predetermined numbers, although an infinite set of pairs. Algebra is even useful in rate problems, such as calculating how the money in your savings account increases because of the interest rate R, such as $Y=X_0+Rt$, where t is elapsed time and X_0 is the initial deposit. With compound interest, things get complicated for algebra, as the rate R is itself a function of time with $Y=X_0+R(t)t$. Now we have a rate of change which itself is changing. Calculus came to the rescue, as Isaac Newton introduced the world to mathematics specifically designed to handle those things that change.
Calculus is among the most important and useful developments of human thought. Even though it is over 300 years old, it is still considered the beginning and cornerstone of modern mathematics. It is a wonderful, beautiful, and useful set of ideas and techniques. You will see the fundamental ideas of this course over and over again in future courses in mathematics as well as in all of the sciences (like physics, biology, social sciences, economics, and engineering). However, calculus is an intellectual step up from your previous mathematics courses. Many of the ideas you will gain in this course are more carefully defined and have both a functional and a graphical meaning. Some of the algorithms are quite complicated, and in many cases, you will need to make a decision as to which appropriate algorithm to use. Calculus offers a huge variety of applications and many of them will be saved for courses you might take in the future.

This course is divided into five learning sections, or units, plus a reference section, or appendix. The course begins with a unit that provides a review of algebra specifically designed to help and prepare you for the study of calculus. The second unit discusses functions, graphs, limits, and continuity. Understanding limits could not be more important, as that topic really begins the study of calculus. The third unit introduces and explains derivatives. With derivatives, we are now ready to handle all of those things that change mentioned above. The fourth unit makes visual sense of derivatives by discussing derivatives and graphs. The fifth unit introduces and explains antiderivatives and definite integrals. Finally, the reference section provides a large collection of reference facts, geometry, and trigonometry that will assist you in solving calculus problems long after the course is over.

Unit 1: Preview and Review

While a first course in calculus can strike you as something new to learn, it is not comparable to learning a foreign language where everything seems different. Calculus still depends on most of the things you learned in algebra, and the true genius of Isaac Newton was to realize that he could get answers for this something new by relying on simple and known things like graphs, geometry, and algebra. There is a need to review those concepts in this unit, where a graph can reinforce the adage that a picture is worth one thousand words. This unit starts right off with one of the most important steps in

mastering problem solving: Have a clear and precise statement of what the problem really is about.

Completing this unit should take you approximately 7 hours.

- Upon successful completion of this unit, you will be able to:
  - approximate a slope of a tangent line from a function given as a graph;
  - approximate the area of an irregular figure by counting inside squares;
  - calculate the slope of the line through two points;
  - write the equation of the line through two points using both slope-intercept and point-slope forms;
  - write the equation of a circle with a given center and radius;
  - evaluate a function at a point, given by a formula, graph, table, or words;
  - evaluate a combination, or a composition, of functions when indicated by the symbols +, - , *, and /;
  - evaluate and graph the elementary functions as well as |x||x| and int(x);
  - state whether a given "if-then" statement is true or false, and justify the answer;
  - state which parts of a mathematical statement are assumptions, or hypotheses, and which are conclusions; and
  - state the contrapositive form of an "if-then" statement.

- 1.1: Preview of Calculus
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.1: Preview of Calculus"URL

    Read this section for an introduction to calculus.

  - 1.1.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.1: Practice Problems"URL

      Work through the odd-numbered problems 1 - 7 on page 5 and page 6. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 1.1.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - The Slope of a Tangent Line; pages 1-2.
      - The Area of a Shape; pages 3-4.
      - Limits; page 4.
      - Differentiation and Integration; page 4.

- 1.2: Lines in the Plane
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.2: Lines in the Plane"URL

Read this section and work through practice problems 1-9. For solutions to the practice problems, see page 15.

- 1.2.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.2: Practice Problems"URL

    Work through the odd-numbered problems 1-29 on pages 10-14. Once you have completed the problem set, check your answers here.

- 1.2.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - The Real Number Line; page 1.
    - The Cartesian Plane; page 2.
    - Increments and Distance between Points in the Plane; pages 2-3.
    - Slope between Points in the Plane; pages 4-6.
    - Equations of Lines; page 6.
    - Two-Point and Slope-Intercept Equations; pages 6-7.
    - Angles between Lines; page 8.
    - Parallel and Perpendicular Lines; pages 8-9.
    - Angles and Intersecting Lines; page 10.

- 1.3: Functions and Their Graphs
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.3: Functions and Their Graphs"URL

    Read this section for an introduction to functions and their graphs. Work through practice problems 1-5. For solutions to the practice problems, see pages 13-14.

- 1.3.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.3: Practice Problems"URL

    Work through the odd-numbered problems 1-23 on pages 8-13. Once you have completed the problem set, check your answers here.

- 1.3.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Definition of a Function; page 1.
    - Function Machines; page 2.
    - Functions Defined by Equations; pages 2-3.
    - Functions Defined by Graphs and Tables of Values; pages 3-4.
    - Creating Graphs of Functions; pages 4-5.
    - Reading Graphs; pages 6-8.

- 1.4: Combinations of Functions

- Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.4: Combinations of Functions"URL

  Read this section on pages 1-11 for an introduction to combinations of functions, then work through practice problems 1-9. For solutions to the practice problems, see pages 18-20.

- 1.4.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.4: Practice Problems"URL

    Work through the odd-numbered problems 1-31 on pages 11-16. Once you have completed the problem set, check your answers here.

- 1.4.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Multiline Definition of Functions; page 1.
    - Wind Chill Index Sample; pages 1-3.
    - Composition of Functions - Functions of Functions; pages 3-4.
    - Shifting and Stretching Graphs; pages 5-6.
    - Iteration of Functions; pages 6-7.
    - Absolute Value and Greatest Integer; pages 7-9.
    - Broken Graphs and Graphs with Holes; pages 10-11.

- 1.5: Mathematical Language
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.5: Mathematical Language"URL

    Read this section on pages 1-5 for an introduction to mathematical language, then work through practice problems 1-4. For the solutions to the practice problems, see pages 7-8.

- 1.5.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 1.5: Practice Problems"URL

    Work through the odd-numbered problems 1-25 on pages 5-7. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 1.5.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Equivalent Statements; page 1.
    - The Logic of "And" and "Or"; page 1.
    - Negation of a Statement; page 2.
    - "If-Then" Statements; pages 2-3.
    - Contrapositive of "If-Then" Statements; page 4.
    - Converse of "If-Then" Statements; pages 4-5.

- Problem Set 1
    - Problem Set 1Quiz
      Take this assessment to see how well you understood these concepts.
        - This assessment **does not count towards your grade**. It is just for practice!
        - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
        - You can take this assessment as many times as you want, whenever you want.

Unit 2: Functions, Graphs, Limits, and Continuity

The concepts of continuity and the meaning of a limit form the foundation for all of calculus. Not only must you understand both of these concepts individually, but you must understand how they relate to each other. They are a kind of Siamese twins in calculus problems, as we always hope they show up together.

A student taking a calculus course during a winter term came up with the best analogy that I have ever heard for tying these concepts together: The weather was raining ice - the kind of weather in which no human being in his right mind would be driving a car. When he stepped out on the front porch to see whether the ice-rain had stopped, he could not believe his eyes when he saw the headlights of an automobile heading down his road, which ended in a dead end at a brick house. When the car hit the brakes, the student's intuitive mind concluded that at the rate at which the velocity was decreasing (assuming continuity), there was no way the car could stop in time and it would hit the house (the limiting value). Oops. He forgot that there was a gravel stretch at the end of the road and the car stopped many feet from the brick house. The gravel represented a discontinuity in his calculations, so his limiting value was not correct.

Completing this unit should take you approximately 19 hours.

- Upon successful completion of this unit, you will be able to:
    - determine the values of one- or two-sided limits for a function given by a graph;
    - use algebraic methods to determine the values of one- and two-sided limits for a function given by a formula or state that the limit "does not exist";
    - state whether a given function is continuous at a point, and use the properties of continuity to find limits and values of related functions;
    - use the Intermediate Value Theorem to determine the number of times a function has a given value;
    - approximate the roots of functions using the Bisection Algorithm;
    - state the epsilon-delta definition of limit; and

        - for a given epsilon, find the required delta graphically and algebraically for linear and quadratic functions.

- 2.1: Tangent Lines, Velocities, and Growth
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.1: Tangent Lines, Velocities, and Growth"URL

      Read this section on pages 1-7 for an introduction to connecting derivatives to quantities we can see in the real world. Work through practice problems 1-4. For the solutions to these practice problems, see page 10-11.

    - 2.1.1: Practice Problems
        - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.1: Practice Problems"URL

          Work through the odd-numbered problems 1-9 on pages 7-9. Once you have completed the problem set, check your answers for the odd-numbered questions here.

    - 2.1.2: Review
        - Before moving on, you should be comfortable with each of these topics:
            - The Slope of a Tangent Line; pages 1-3.
            - Average Velocity and Instantaneous Velocity; pages 3-5.
            - Average Population Growth Rate and Instantaneous Population Growth Rate; pages 5-7.
- 2.2: The Limit of a Function
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.2: The Limit of a Function"URL

      Read this section on pages 1-7 for an introduction to connecting derivatives to quantities we can see in the real world. Work through practice problems 1-4. For solutions to these practice problems, see page 10.

    - 2.2.1: Practice Problems
        - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.2: Practice Problems"URL

          Work through the odd-numbered problems 1-19 on pages 7-9. Once you have completed the problem set, check your answers for the odd-numbered questions here.

    - 2.2.2: Review
        - Before moving on, you should be comfortable with each of these topics:
            - Informal Notion of a Limit; pages 1-3.
            - Algebra Method for Evaluating Limits; pages 4-6.

- Table Method for Evaluating Limits; pages 4-6.
- Graph Method for Evaluating Limits; pages 4-6.
- One-Sided Limits; pages 6-7.

- 2.3: Properties of Limits
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.3: Properties of Limits"URL

    Read this section on pages 1-8 to learn about the properties of limits. Work through practice problems 1-6. For the solutions to these problems, see page 14.

  - RootMath: "Solving Limits (Rationalization)"Page

    Watch this video on finding limits algebraically. Be warned that removing x−4x−4 from the numerator and denominator in Step 4 of this video is only legal inside this limit. The function x−4x−4x−4x−4 is not defined at x=4x=4; however, when xx is not 4, it simplifies to 1. Because the limit as xx approaches 4 depends only on values of xx different from 4, inside that limit x−4x−4x−4x−4 and 1 are interchangeable. Outside that limit, they are not! However, this kind of cancellation is a key technique for finding limits of algebraically complicated functions.

  - Khan Academy: "Calculating Slope of Tangent Line Using Derivative Definition"Page

    Watch this video on limits as the slopes of tangent lines.

    An earlier Khan Academy video (not used in this course) defined the limit that gives the slope of the tangent line to a curve as y=f(x)y=f(x) at a point x=ax=a and called it the derivative of f(x)f(x) at aa. The text will introduce this term in Unit 3.

  - 2.3.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.3: Practice Problems"URL

      Work through the odd-numbered problems 1-21 on pages 9-14. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 2.3.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - Main Limit Theorem; page 1.
      - Limits by Substitution; page 2.
      - Limits of Combined or Composed Functions; pages 2-4.
      - Tangent Lines as Limits; page 4 and page 5.
      - Comparing the Limits of Functions; page 5 and page 6.
      - Showing that a Limit Does Not Exist; pages 6-8.
- Problem Set 2

- Problem Set 2Quiz

Take this assessment to see how well you understood these concepts.

- This assessment **does not count towards your grade**. It is just for practice!
- You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
- You can take this assessment as many times as you want, whenever you want.

- 2.4: Continuous Functions
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.4: Continuous Functions"URL

    Read this section on pages 1-11 for an introduction to what we mean when we say a function is continuous. Work through practice problems 1 and 2. For solutions to these problems, see page 16.

  - 2.4.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.4: Practice Problems"URL

      Work through the odd-numbered problems 1-23 on pages 12-15. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 2.4.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - Definition and Meaning of Continuous; page 1.
      - Graphic Meaning of Continuity; pages 1-4.
      - The Importance of Continuity; page 5.
      - Combinations of Continuous Functions; pages 5-6.
      - Which Functions Are Continuous?; pages 6-8.
      - Intermediate Value Property; page 8 and page 9.
      - Bisection Algorithm for Approximating Roots; pages 9-11.
- 2.5: Definition of a Limit
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.5: Definition of a Limit"URL

    Read this section on pages 1-11 to learn how a limit is defined. Work through practice problems 1-6. For solutions to these problems, see pages 14-16.

  - Khan Academy: "Epsilon-Delta Limit Definition 1"Page

    Watch this video to learn the epsilon-delta definition of a limit.

- 2.5.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 2.5: Practice Problems"URL

    Work through the odd-numbered problems 1-23 on pages 12-14. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 2.5.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Intuitive Approach to Defining a Limit; pages 1-7.
    - The Formal Definition of a Limit; pages 7-10.
    - Two Limit Theorems; pages 10-11.

- Problem Set 3
  - Problem Set 3Quiz

    Take this assessment to see how well you understood these concepts.

    - This assessment **does not count towards your grade**. It is just for practice!
    - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
    - You can take this assessment as many times as you want, whenever you want.

Unit 3: Derivatives

In this unit, we start to see calculus become more visible when abstract ideas such as a derivative and a limit appear as parts of slopes, lines, and curves. Then, there are circles, ellipses, and parabolas that are even more geometric, so what was previously an abstract concept can now be something we can see. Nothing makes calculus more tangible than to recognize that the first derivative of an automobile's position is its velocity and the second derivative of that position is its acceleration. We are at the very point that started Isaac Newton on his quest to master this mathematics, what we now call calculus, when he recognized that the second derivative was precisely what he needed to formulate his Second Law of Motion $F=MAF=MA$, where FF is the force on any object, MM is its mass, and AA is the second derivative of its position. Thus, he could connect all the variables of a moving object mathematically, including its acceleration, velocity, and position, and he could explain what really makes motion happen.

Completing this unit should take you approximately 42 hours.

- Upon successful completion of this unit, you will be able to:
  - state the definition of a derivative of a function $f(x)$f(x);
  - recognize and use the common equivalent notations for the derivative;

- state the graph and rate meanings of a derivative;
- estimate a tangent line slope and instantaneous rate of change from the graph of a function;
- write the equation of the line tangent to the graph of a function $f(x)$f(x);
- calculate the derivatives of the elementary functions;
- calculate second and higher derivatives and state what they measure;
- differentiate compositions of functions using the chain rule;
- calculate the derivatives of functions given as parametric equations and interpret their meanings geometrically and physically;
- state whether a function, given by a graph or formula, is continuous or differentiable at a point or on an interval;
- solve related rate problems using derivatives;
- approximate the solutions of equations by using derivatives and Newton's method;
- approximate the values of difficult functions by using derivatives;
- calculate the differential of a function using derivatives and show what the differential represents on a graph; and
- calculate the derivatives of really difficult functions by using the methods of implicit differentiation and logarithmic differentiation.

- 3.1: Introduction to Derivatives
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.1: Introduction to Derivatives"URL

    Read this section on pages 1-5 to lay the groundwork for introducing the concept of a derivative. Work through practice problems 1-5. For solutions to these problems, see pages 8-9.

  - 3.1.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.1: Practice Problems"URL

      Work through the odd-numbered problems 1-17 on pages 5-7. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 3.1.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - Slopes of Tangent Lines; pages 1-2.
      - Tangents to $y=x2$y=x2; pages 2-5.
- 3.2: The Definition of a Derivative
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.2: The Definition of a Derivative"URL

Read this section on pages 1-10 to understand the definition of a derivative. Work through practice problems 1-8. For solutions to these problems, see pages 14-15.

- 3.2.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.2: Practice Problems"URL

    Work through the odd-numbered problems 1-37 on pages 11-14. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 3.2.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Formal Definition of a Derivative; pages 1-2.
    - Calculations Using the Definition; pages 2-6.
    - Tangent Line Formula; page 4.
    - sinsin and coscos Examples; pages 4-5.
    - Interpretations of the Derivative; pages 6-8.
    - A Useful Formula: D(xn)D(xn); pages 8-10.
    - Important Definitions, Formulas, and Results for the Derivative, Tangent Line Equation, and Interpretations of f′(x)f′(x); page 10.

- 3.3: Derivatives, Properties and Formulas
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.3: Derivatives, Properties and Formulas"URL

    Read this section on pages 1-9 to understand the properties of derivatives. Work through practice problems 1-11. For solutions to these problems, see pages 16-17.

  - Khan Academy: "Applying the Product Rule for Derivatives"Page

    Watch this video on the product rule for differentiation.

- 3.3.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.3: Practice Problems"URL

    Work through the odd-numbered problems 1-55 on pages 10-15. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 3.3.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Which Functions Have Derivatives?; pages 1-3.
    - Derivatives of Elementary Combination of Functions; pages 3-6.

- Using the Differentiation Rules; pages 7-8.
- Evaluative a Derivative at a Point; page 9.
- Important Results for Differentiability and Continuity; page 9.

- 3.4: Derivative Patterns
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.4: More Differentiation Problems"URL

    Read this section on pages 1-9 to learn about patterns of derivatives. Work through practice problems 1-8. For solutions to these problems, see pages 12-14.

  - 3.4.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.4: Practice Problems"URL

      Work through the odd-numbered problems 1-47 on pages 9-14. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 3.4.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - A Power Rule for Functions: $D(fn(x))$D(fn(x)): To review this topic, focus on pages 1 and 2.
      - Derivatives of Trigonometric and Exponential Functions: To review this topic, focus on pages 3-6.
      - Higher Derivatives - Derivatives of Derivatives: To review this topic, focus on pages 6-7.
      - Bent and Twisted Functions: To review this topic, focus on pages 7-8.
      - Important Results for Power Rule of Functions and Derivatives of Trigonometric and Exponential Functions; page 9.
- 3.5: The Chain Rule
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.5: The Chain Rule"URL

    Read this section on pages 1-7 to learn about the Chain Rule. Work through practice problems 1-8. For solutions to these problems, see pages 11-12.

  - Khan Academy: "Chain Rule Introduction" and "Chain Rule Definition and Example"Page

    Watch these videos for an introduction to the chain rule for differentiation and examples of the chain rule for differentiation.

- 3.5.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.5: Practice Problems"URL

    Work through the odd-numbered problems 1-83 on pages 7-11. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 3.5.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Chain Rule for Differentiating a Composition of Functions; page 1.
    - The Chain Rule Using Leibnitz Notation Form; page 2.
    - The Chain Rule Composition Form; pages 2-5.
    - The Chain Rule and Tables of Derivatives; pages 5-6.
    - The Power Rule for Functions; page 7.

- Problem Set 4
  - Problem Set 4Quiz

    Take this assessment to see how well you understood these concepts.

    - This assessment **does not count towards your grade**. It is just for practice!
    - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
    - You can take this assessment as many times as you want, whenever you want.

- 3.6: Some Applications of the Chain Rule
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.6: Some Applications of the Chain Rule"URL

    Read this section on pages 1-8 to learn how to apply the Chain Rule. Work through practice problems 1-8. For solutions to these problems, see pages 13-14.

- 3.6.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.6: Practice Problems"URL

    Work through the odd-numbered problems 1-49 on pages 9-11. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 3.6.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Derivatives of Logarithms; pages 1-2.

        - Derivative of axax; pages 2-3.
        - Applied Problems; pages 3-5.
        - Parametric Equations; pages 5-6.
        - Speed; page 8.
- 3.7: Related Rates
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.7: Related Rates: An Application of Derivatives"URL

      Read this section on pages 1-7 to learn to connect derivatives to the concept of the rate at which things change. Work through practice problems 1-3. For solutions to these problems, see pages 12-13.

    - 3.7.1: Practice Problems
        - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.7: Practice Problems"URL

          Work through the odd-numbered problems 1-21 on pages 8-12. Once you have completed the problem set, check your answers for the odd-numbered questions here.

    - 3.7.2: Review
        - Before moving on, you should be comfortable with each of these topics:
            - The Derivative as a Rate of Change; pages 1-7.
- 3.8: Newton's Method for Finding Roots
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's Contemporary Calculus: "Section 3.8: Newton's Method for Finding Roots"URL

      Read this section on pages 1-8. Work through practice problems 1-6. For solutions to these problems, see pages 10-11.

    - Massachusetts Institute of Technology: Christine Breiner's "Using Newton's Method"Page

      Watch this video on Newton's method.

    - 3.8.1: Practice Problems
        - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.8: Practice Problems"URL

          Work through the odd-numbered problems 1-21 on pages 8-9. Once you have completed the problem set, check your answers for the odd-numbered questions here.

    - 3.8.2: Review

- Before moving on, you should be comfortable with each of these topics:
    - Newton's Method Using the Tangent Line; pages 1-3.
    - The Algorithm for Newton's Method; pages 3-5.
    - Iteration; page 5.
    - What Can Go Wrong with Newton's Method?; pages 5-6.
    - Chaotic Behavior and Newton's Method; pages 6-8.

- 3.9: Linear Approximation and Differentials
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.9: Linear Approximation and Differentials"URL

      Read this section on pages 1-10 to learn how linear approximation and differentials are connected. Work through practice problems 1-10. For the solutions to these problems, see pages 14-15.

    - RootMath: "Linear Approximation"Page

      Watch this video on linear approximation and differentials.

- 3.9.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.9: Practice Problems"URL

      Work through the odd-numbered problems 1-19 on pages 10-13. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 3.9.2: Review
    - Before moving on, you should be comfortable with each of these topics:
        - Linear Approximation and Its Process; pages 1-4.
        - Applications of Linear Approximation to Measurement Error; pages 4-6.
        - Relative Error and Percentage Error; pages 6-7.
        - The Differential of a Function; pages 7-8.
        - The Linear Approximation Error; pages 8-10.

- 3.10: Implicit and Logarithmic Differentiation
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.10: Implicit and Logarithmic Differentiation"URL

      Read this section on pages 1-5 to learn about implicit and logarithmic differentiation. Work through practice problems 1-6. For solutions to these problems, see pages 8-9.

    - Khan Academy: "Implicit Differentiation" and "Derivative of x^(x^x)"Page

      Watch these videos on implicit and logarithmic differentiation.

- 3.10.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 3.10: Practice Problems"URL

    Work through the odd-numbered problems 1-55 on pages 5-8. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 3.10.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Implicit Differentiation; pages 1-3.
    - Logarithmic Differentiation; pages 3-5.

- Problem Set 5
  - Problem Set 5Quiz

    Take this assessment to see how well you understood these concepts.

    - This assessment **does not count towards your grade**. It is just for practice!
    - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
    - You can take this assessment as many times as you want, whenever you want.

Unit 4: Derivatives and Graphs

A visual person should find this unit extremely helpful in understanding the concepts of calculus, as a major emphasis in this unit is to display those concepts graphically. That allows us to see what, so far, we could only imagine. Graphs help us to visualize ideas that are hard enough to conceptualize - like limits going to infinity but still having a finite meaning, or asymptotes - lines that approach each other but never quite get there.

Graphs can also be used in a kind of reverse by displaying something for which we should take another mathematical look. It is hard enough to imagine a limit going to infinity, and therefore never quite getting there, but the graph can tell us that it has a finite value, when it finally does get there, so we had better take a serious look at it mathematically.

Completing this unit should take you approximately 29 hours.

- Upon successful completion of this unit, you will be able to:
  - state whether a given point on a graph is a global/local maximum/minimum;
  - find critical points and extreme values (max/min) of functions by using derivatives;

- determine the values of a function guaranteed to exist by Rolle's Theorem and by the Mean Value Theorem;
- use the graph of $f(x)$ to sketch the shape of the graph of $f(x)$;
- use the values of $f'(x)$ to sketch the graph of $f(x)$ and state whether $f(x)$ is increasing or decreasing at a point;
- use the values of $f''(x)$ to determine the concavity of the graph of $f(x)$;
- use the graph of $f(x)$ to determine if $f''(x)$ is positive, negative, or zero;
- solve maximum and minimum problems by using derivatives;
- restate in words the meanings of the solutions to applied problems, attaching the appropriate units to an answer;
- determine asymptotes of a function by using limits; and
- determine the values of indeterminate form limits by using derivatives and L'Hopital's Rule.

- 4.1: Finding Maximums and Minimums
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.1: Finding Maximums and Minimums"URL

    Read this section on pages 1-9 to learn about maximums, minimums, and extreme values for functions. Work through practice problems 1-5. For solutions to these problems, see pages 13-14.

  - 4.1.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.1: Practice Problems"URL

      Work through the odd-numbered problems 1-43 on pages 9-13. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 4.1.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - Methods for Finding Maximums and Minimums; page 1.
      - Terminology: Global Maximum, Local Maximum, Maximum Point, Global Minimum, Local Minimum, Global Extreme, and Local Extreme; page 2.
      - Finding Maximums and Minimums of a Function; pages 3-5.
      - Is $f(a)$ a Maximum, Minimum, or Neither?; page 5.
      - Endpoint Extremes; pages 5-7.
      - Critical Numbers; page 7.
      - Which Functions Have Extremes?; pages 7-8.
      - Extreme Value Theorem; pages 8-9.
- 4.2: The Mean Value Theorem and Its Consequences

- Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.2: The Mean Value Theorem and Its Consequences"URL

  Read this section on pages 1-6 to learn about the Mean Value Theorem and its consequences. Work through practice problems 1-3. For solutions to these problems, see pages 9 and 10.

- 4.2.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.2: Practice Problems"URL

    Work through the odd-numbered problems 1-35 on pages 6-9. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 4.2.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Rolle's Theorem; pages 1-2.
    - The Mean Value Theorem; pages 2-4.
    - Consequences of the Mean Value Theorem; pages 4-6.

- 4.3: The First Derivative and the Shape of a Function f(x)
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.3: The First Derivative and the Shape of f"URL

    Read this section on pages 1-8 to learn how the first derivative is used to determine the shape of functions. Work through practice problems 1-9. For the solution to these problems, see pages 10-12.

  - RootMath: "First Derivative Test, Example 2"Page

    Watch both parts of this video on the first derivative test.

- 4.3.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.3: Practice Problems"URL

    Work through the odd-numbered problems 1-29 on pages 8-10. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 4.3.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Definitions of the Function; page 1.
    - First Shape Theorem; pages 2-4.

        - Second Shape Theorem; pages 4-7.
        - Using the Derivative to Test for Extremes; pages 7-8.
- 4.4: The Second Derivative and the Shape of a Function f(x)
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.4: Second Derivative and the Shape of f"URL

      Read this section on pages 1-6 to learn how the second derivative is used to determine the shape of functions. Work through practice problems 1-9. For solutions to these problems, see pages 8-9.

    - Khan Academy: "Concavity, Concave upwards and Concave downwards Intervals"Page

      Watch this video on the second derivative test. This video describes a way to identify critical points as minima or maxima other than the first derivative test, using the second derivative.

    - RootMath: "Concavity and the Second Derivative"Page

      Watch this video, which works through an example of the second derivative test.

    - 4.4.1: Practice Problems
        - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.4: Practice Problems"URL

          Work through the odd-numbered problems 1-17 on pages 6-8. Once you have completed the problem set, check your answers for the odd-numbered questions here.

    - 4.4.2: Review
        - Before moving on, you should be comfortable with each of these topics:
            - Concavity; pages 1-2.
            - The Second Derivative Condition for Concavity; pages 2-3.
            - Feeling the Second Derivative: Acceleration Applications; pages 3-4.
            - The Second Derivative and Extreme Values; pages 4-5.
            - Inflection Points; pages 5-6.
- Problem Set 6
    - Problem Set 6Quiz

      Take this assessment to see how well you understood these concepts.

        - This assessment **does not count towards your grade**. It is just for practice!
        - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!

- You can take this assessment as many times as you want, whenever you want.

- 4.5: Applied Maximum and Minimum Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.5: Applied Maximum and Minimum Problems"URL

    Read this section on pages 1-6 to learn how to apply previously learned principles to maximum and minimum problems. Work through practice problems 1-3. For solutions to these problems, see pages 15-16. There is no review for this section; instead, make sure to study the problems carefully to become familiar with applied maximum and minimum problems.

  - Khan Academy: "Minimizing Sum of Squares"Page

    Watch this video on optimization.

  - Massachusetts Institute of Technology: Christine Breiner's "Minimum Triangle Area"Page

    Watch this video on optimization.

  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.5: Practice Problems"URL

    Work through the odd-numbered problems 1-33 on pages 6-15. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 4.6: Infinite Limits and Asymptotes
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.6: Infinite Limits and Asymptotes"URL

    Read this section on pages 1-10 to learn how to use and apply infinite limits to asymptotes. Work through practice problems 1-8. For solutions to these problems, see pages 13-14.

  - 4.6.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.6: Practice Problems"URL

      Work through the odd-numbered problems 1-59 on pages 10-12. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 4.6.2: Review

- Before moving on, you should be comfortable with each of these topics:
  - Limits as xx Approaches Infinity; pages 1-4.
  - Using Calculators to Find Limits as xx Goes to Infinity; page 5.
  - The Limit Is Infinite; pages 5-6.
  - Horizontal Asymptotes; pages 6-7.
  - Vertical Asymptotes; pages 7-8.
  - Other Asymptotes as xx Approaches Infinity; pages 8-9.
  - Definition of limx→∞x=klimx→∞x=k; pages 9-10.

- 4.7: L'Hopital's Rule
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.7: L'Hopital's Rule"URL

    Read this section on pages 1-6 to learn how to use and apply L'Hopital's Rule. Work through practice problems 1-3. For solutions to these problems, see page 8.

  - Khan Academy: "Introduction to L'Hopital's Rule"Page

    Watch this video for an introduction to L'Hopital's Rule.

  - 4.7.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 4.7: Practice Problems"URL

      Work through the odd-numbered problems 1-29 on pages 6-7. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 4.7.2: Review
    - Before moving on, you should be comfortable with each of these topics found in the resources linked to above:
      - A Linear Example; page 1.
      - 0/0 Form of L'Hopital's Rule; page 2.
      - Strong Version of L'Hopital's Rule; pages 2-3.
      - Which Function Grows Faster?; page 4.
      - Other Indeterminate Forms; pages 4-6.

- Problem Set 7
  - Problem Set 7Quiz

    Take this assessment to see how well you understood these concepts.

    - This assessment **does not count towards your grade**. It is just for practice!
    - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
    - You can take this assessment as many times as you want, whenever you want.

Unit 5: The Integral

While previous units dealt with differential calculus, this unit starts the study of integral calculus. As you may recall, differential calculus began with the development of the intuition behind the notion of a tangent line. Integral calculus begins with understanding the intuition behind the notion of an area. In fact, we will be able to extend the notion of the area and apply these more general areas to a variety of problems. This will allow us to unify differential and integral calculus through the Fundamental Theorem of Calculus. Historically, this theorem marked the beginning of modern mathematics and is extremely important in all applications.

Completing this unit should take you approximately 32 hours.

- Upon successful completion of this unit, you will be able to:
  - use sigma notation to represent sums;
  - approximate areas by Riemann sums;
  - translate an area under a curve into a definite integral
  - evaluate definite integrals geometrically using graphs of functions;
  - determine whether a given function is integrable;
  - find antiderivatives by changing the variable and using tables;
  - use the Fundamental Theorem of Calculus to evaluate definite integrals
  - differentiate integrals;
  - solve applied problems that involve generalized area, that is, distance, work, and so forth;
  - find an area between two curves; and
  - find the average (mean) value of a function.

- 5.1: Introduction to Integration
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.1: Introduction to Integration"URL

    Read this section on pages 1-7 to learn about area. Work through practice problems 1-9. For solutions to these problems, see page 10.

- 5.1.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.1: Practice Problems"URL

    Work through the odd-numbered problems 1-15 on pages 8-9. Once you have completed the problem set, check your answers for the odd-numbered questions against here.

- 5.1.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Area; pages 1-4.
    - Applications of Area like Distance and Total Accumulation; pages 5-7.

- 5.2: Sigma Notation and Riemann Sums
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.2: Sigma Notation and Riemann Sums"URL

    Read this section on pages 1-10 to learn about area. Work through practice problems 1-9. For solutions to these problems, see pages 15-16.

  - 5.2.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.2: Practice Problems"URL

      Work through the odd-numbered problems 1-61 on pages 10-15. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 5.2.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - Sigma Notation; pages 1-2.
      - Sums of Areas of Rectangles; pages 3-4.
      - Area under a Curve - Riemann Sums; pages 5-8.
      - Two Special Riemann Sums - Lower and Upper Sums; pages 9-10.
- 5.3: The Definite Integral
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.3: The Definite Integral"URL

    Read this section on pages 1-6 to learn about the definite integral and its applications. Work through practice problems 1-6. For solutions to these problems, see page 11.

  - 5.3.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.3: Practice Problems"URL

      Work through the odd-numbered problems 1-29 on pages 6-10. Once you have completed the problem set, check your answers for the odd-numbered questions here.

  - 5.3.2: Review
    - Before moving on, you should be comfortable with each of these topics:
      - The Definition of the Definite Integral; pages 1-3.
      - Definite Integrals of Negative Functions; pages 3-5.
      - Units for the Definite Integral; pages 5-6.
- 5.4: Properties of the Definite Integral

- Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.4: Properties of the Definite Integral"URL

  Read this section on pages 1-8 to learn about properties of definite integrals and how functions can be defined using definite integrals. Work through practice problems 1-5. For solutions to these problems, see page 11.

- 5.4.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.4: Practice Problems"URL

    Work through the odd-numbered problems 1-51 on pages 9-11. Once you have completed the problem set, check your answers for the odd-numbered questions against here.

- 5.4.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Properties of the Definite Integral; pages 1-2.
    - Properties of Definite Integrals of Combinations of Functions; pages 3-5.
    - Functions Defined by Integrals; pages 5-6.
    - Which Functions Are Integrable?; pages 6-7.
    - A Nonintegrable Function; page 8.

- 5.5: Areas, Integrals, and Antiderivatives
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.5: Areas, Integrals, and Antiderivatives"URL

    Read this section on pages 1-6 to learn about the relationship among areas, integrals, and antiderivatives. Work through practice problems 1-5. For solutions to these problems, see pages 10-11.

- 5.5.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.5: Practice Problems"URL

    Work through the odd-numbered problems 1-25 on pages 7-9. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 5.5.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Area Functions as an Antiderivative; pages 1-2.
    - Using Antiderivatives to Evaluate Definite Integrals; pages 2-4.
    - Integrals, Antiderivatives, and Applications; pages 4-6.

- 5.6: The Fundamental Theorem of Calculus

- Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.6: The Fundamental Theorem of Calculus"File

  Read this section on pages 1-9 to see the connection between derivatives and integrals. Work through practice problems 1-5. For solutions to these problems, see pages 14-15.

- 5.6.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.6: Practice Problems"URL

    Work through the odd-numbered problems 1-67 on pages 10-13. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 5.6.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Antiderivatives; pages 1-3.
    - Evaluating Definite Integrals; pages 4-5.
    - Steps for Calculus Application Problems; pages 6-8.
    - Leibnitz's Rule for Differentiating Integrals; page 9.

- Problem Set 8
  - Problem Set 8Quiz

    Take this assessment to see how well you understood these concepts.

    - This assessment **does not count towards your grade**. It is just for practice!
    - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
    - You can take this assessment as many times as you want, whenever you want.

- 5.7: Finding Antiderivatives
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.7: Finding Antiderivatives"URL

    Read this section on pages 1-9 to see how one can (sometimes) find an antiderivative. In particular, we will discuss the change of variable technique. Change of variable, also called substitution or u-substitution (for the most commonly-used variable), is a powerful technique that you will use time and again in integration. It allows you to simplify a complicated function to show how basic rules of integration apply to the function. Work through practice problems 1-4. For solutions to these problems, see pages 12-13.

- RootMath: "Integration: U-Substitution"Page

  Watch these videos on change of variable, also called substitution or u-substitution.

- 5.7.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.7: Practice Problems"URL

    Work through the odd-numbered problems 1-69 on pages 10-12. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 5.7.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Indefinite Integrals and Antiderivatives; page 1.
    - Properties of Antiderivatives (Indefinite Integrals); pages 2-3.
    - Antiderivatives of More Complicated Functions; pages 3-4.
    - Getting the Constant Right; pages 4-5.
    - Making Patterns More Obvious - Changing Variables; pages 5-8.
    - Changing the Variables and Definite Integrals; pages 8-9.
    - Special Transformations - Antiderivatives of $\sin^2(x)$ and $\cos^2(x)$; page 9.

- 5.8: First Application of Definite Integral
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.8: First Application of Definite Integrals"URL

    Read this section on pages 1-8 to see how some applied problems can be reformulated as integration problems. Work through practice problems 1-4. For solutions to these problems, see pages 10-11.

- 5.8.1: Practice Problems
  - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.8: Practice Problems"URL

    Work through the odd-numbered problems 1-41 on pages 8-10. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 5.8.2: Review
  - Before moving on, you should be comfortable with each of these topics:
    - Area between Graphs of Two Functions; pages 1-4.
    - Average (Mean) Value of a Function; pages 4-6.
    - A Definite Integral Application - Work; pages 6-8.

- 5.9: Using Tables to Find Antiderivatives
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.9: Using Tables to Find Antiderivatives"URL

      Read this section on pages 1-3 to learn how to use tables to find antiderivatives. See the Calculus Reference Facts for the table of integrals mentioned in the reading. Work through practice problems 1-5. For solutions to these problems, see page 6.

- 5.9.1: Practice Problems
    - Washington State Board for Community and Technical Colleges: Dale Hoffman's "Contemporary Calculus, Section 5.9: Practice Problems"URL

      Work through the odd-numbered problems 1-55 on pages 4-5. Once you have completed the problem set, check your answers for the odd-numbered questions here.

- 5.9.2: Review
    - Before moving on, you should be comfortable with each of these topics:
        - Table of Integrals; pages 1-3.
        - Using Recursive Formulas; page 3.

- Problem Set 9
    - Problem Set 9Quiz

      Take this assessment to see how well you understood these concepts.

        - This assessment **does not count towards your grade**. It is just for practice!
        - You will see the correct answers when you submit your answers. Use this to help you study for the final exam!
        - You can take this assessment as many times as you want, whenever you want.

- Appendix

By reviewing this course, you will have an invaluable list of references to assist you in solving future calculus problems after this course has ended. It is a standard experience, when solving calculus problems on your own, to react to the new problem with the following: "We did not solve that kind of problem in the course." Ah, but we did, in that the new problem is often a combination, or composition, of two problem types that were covered.

The course could not cover all possible trigonometric functions you will encounter. If you encounter a need for the derivative of $\tan(x)$tan(x), it is sufficient to recall that $\tan(x)=\sin(x)\cos(x)$tan(x)=sin(x)cos(x) and that sine and cosine were covered. You

can eventually become so good at this that future calculus problems can almost seem to be little more than plugging into formulas.

Engineering students who have to take several courses that involve the use of calculus are noted for having a Table of Integrals on their hip wherever they go, such as this one posted on Wikipedia.

Unit 1: Analytic Geometry *Most of the material in this unit will be review. However, the notions of points, lines, circles, distance, and functions will be central in everything that follows. Lines are basic geometric objects which will be of great importance in the study of differential calculus in the study of tangent lines and linear approximations.*

We will also take a look at the practical uses of mathematical functions. This course will use mathematical models, or structures, that predict practical situations in order to describe and study a number of real-life problems and situations. They are essential to the development of every major business and every scientific field in the modern world.

Unit 1 Time Advisory
This unit should take you approximately 9.5 hours to complete.

☐ Subunit 1.1: 2.5 hours

☐ Subunit 1.2: 2 hours

☐ Subunit 1.3: 3 hours

☐ Subunit 1.4: 2 hours

Unit1 Learning Outcomes
Upon successful completion of this unit, the student will be able to:
- Create and analyze graphs of points, lines, and circles in a plane. - Define and identify the domain, range, and graph of a function. - Define and identify one-to-one, onto, and linear functions. - Analyze and graph transformations of functions, such as shifts and dilations.

1.1 Lines - Reading: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Section 1.1: Lines" Link: Whitman College: David Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Section 1.1: Lines" (PDF)

Instructions: Please click on the link above and read Section 1.1 (pages 14-17) in its entirety. Working with lines should be familiar to you, and this section serves as a review of the notions of points, lines, slope, intercepts, and graphing.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Exercises 1.1: Problems 1-18"** Link: Whitman College: David

Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Exercises 1.1: Problems 1–18" (PDF)

Instructions: Please click on the above link and work through problems 1-18. When you are done, check your answers against "Appendix A: Answers".

Completing this assignment should take you about two hours to complete.

1.2 Distance between Two Points, Circles - Reading: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Section 1.2: Distance Between Two Points; Circles" Link: Whitman College: David Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Section 1.2: Distance Between Two Points; Circles" (PDF)

Instructions: Please click on the link above and read Section 1.2 (pages 19 and 20) in its entirety. This reading reviews the notions of distance in the plane and the equations and graphs of circles.

This reading should take approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Exercises 1.2: Problems 1, 2, 6"** Link: Whitman College: David Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Exercises 1.2: Problems 1, 2, 6" (PDF)

  Instructions: Please click on the above link and work through problems 1, 2, and 6 in Exercises 1.2. When you are done, to check your answers against "Appendix A: Answers".

  This assignment should take you approximately 30 minutes to complete.

- **Assignment: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Marc Renault and Molly M. Cow's "Distance" Module and Dan Reich's "Equations of Circles II" Module** Link: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Marc Renault and Molly M. Cow's "Distance" Module and Dan Reich's "Equations of Circles II" Module

  Instructions: Please click on the link above and click on the number 5 next to "Distance" to launch the first module and complete problems 1-5. Then return to the index and click on the

number 8 next to "Circles II" to launch the second module and complete problems 1-5. If at any time the problem set becomes too easy for you, feel free to move forward.

Completing this assignment should take you approximately one hour to complete.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.3 Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Section 1.3: Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Section 1.3: Functions" (PDF)

Instructions: Please click on the link above and read Section 1.3 (pages 20-24) in its entirety. This reading reviews the notion of functions, linear functions, domain, range, and dependent and independent variables.

This reading should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Exercises 1.3: Problems 1-16"** Link: Whitman College: David Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Exercises 1.3: Problems 1-16" (PDF)

  Instructions: Please click on the link above and work through problems 1-16. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately two hours to complete.

1.4 Shifts and Dilations - Reading: Whitman College: David Guichard's Calculus: Chapter 1: Analytic Geometry: "Section 1.4: Shifts and Dilations" Link: Whitman College: David Guichard's *Calculus:* Chapter 1: Analytic Geometry: "Section 1.4: Shifts and Dilations" (PDF)

Instructions: Please click on the link above and read Section 1.4 (pages 25-28) in its entirety. This reading will review graph transformations associated with some basic ways of manipulating functions like like shifts and dilations.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Lavanya Myneni and Molly M. Cow's "Recognizing Algebraic Functions" Module** Link: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Lavanya Myneni and Molly M. Cow's "Recognizing Algebraic Functions" Module

  Instructions: Please click on the link above and select the "Index" button. Click on the number 18 next to "Transforming Graphs" to launch the module and complete problems 1-15. If at any time the problem set becomes too easy for you, feel free to move forward.

  This assignment should take you approximately an hour and 30 minutes to complete.

Unit 2: Instantaneous Rate of Change: The Derivative *In this unit, you will study the instantaneous rate of change of a function. Motivated by this concept, you will develop the notion of limits, continuity, and the derivative. The limit asks the question, "What does the function do as the independent variable becomes closer and closer to a certain value?" In simpler terms, the limit is the "natural tendency" of a function. The limit is incredibly important due to its relationship to the derivative, the integral, and countless other key mathematical concepts. A strong understanding of limits is essential to the field of mathematics.*

A derivative is a description of how a function changes as its input varies. In the case of a straight line, this description is the same at every point, which is why we can describe the slope of an entire function when it is linear. You can also describe the slope of nonlinear functions. The slope, however, will not be constant; it will change as the independent variable changes.

Unit 2 Time Advisory
This unit should take you approximately 16.5 hours to complete.

☐ Subunit 2.1: 2.5 hours

☐ Subunit 2.2: 1 hour

☐ Subunit 2.3: 6.75 hours
☐ Sub-subunit 2.3.1: 6 hours

☐ Sub-subunit 2.3.2: 0.75 hours

☐ Subunit 2.4: 2 hours

☐ Subunit 2.5: 4.25 hours

☐ Introduction: 1 hour

☐ Sub-subunit 2.5.1: 2 hours

☐ Sub-subunit 2.5.2: 1 hour

☐ Sub-subunit 2.5.3: 0.25 hours

Unit2 Learning Outcomes
Upon the successful completion of this unit, the student will be able to:
- Define and calculate limits and one-sided limits. - Identify vertical asymptotes. - Define continuity and determine whether a function is continuous. - State and apply the Intermediate

Value Theorem. - State the Squeeze Theorem and use it to calculate limits. - Calculate limits at infinity and identify horizontal asymptotes. - Calculate limits of rational and radical functions. - State the epsilon-delta definition of a limit and use it in simple situations to show a limit exists. - Draw a diagram to explain the tangent line problem. - State several different versions of the limit definition of the derivative and use multiple notations for the derivative. - Describe the derivative as a rate of change and give some examples of its application, such as velocity. - Calculate simple derivatives using the limit definition.

2.1 The Slope of a Function *Note: In this section, you will look at the first of two major problems at the heart of calculus: the tangent line problem. This intellectual exercise demonstrates the origins of derivatives for nonlinear functions.*

- **Reading: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.1: The Slope of a Function"** Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.1: The Slope of a Function" (PDF)

  Instructions: Please click on the link above and read Section 2.1 (pages 29-33) in its entirety. You will be introduced to the notion of a derivative through studying a specific example. The example will also reveal the necessity of having a precise definition for the limit of a function.

  This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 1: Rate of Change"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 1: Rate of Change" (YouTube)

  Instructions: Please click on the link above and watch the entire video (51:33). Lecture notes are available here. In this lecture, Professor Jerison introduces the derivative as the rate of change of a function, or the slope of the tangent line to a function at a point.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.1: Problems 1-6"** Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.1: Problems 1-6" (PDF)

  Instructions: Please click on the link above and work through problems 1-6. When you are done, check your answers against "Appendix A: Answers".

This assignment should take you approximately 30 minutes to complete.

2.2 An Example - Reading: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.2: An Example" Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.2: An Example" (PDF)

Instructions: Please click on the link above and read Section 2.2 (pages 34-36) in its entirety. This reading discusses the derivative in the context of studying the velocity of a falling object.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.2: Problems 1-3"** Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.2: Problems 1-3" (PDF)

  Instructions: Please click on the link above and work through problems 1-3. When you are done, check your answers against "Appendix A: Answers".

This assignment should take you approximately 30 minutes to complete.

2.3 Limits *In this section, you will take a close look at a concept that you have used intuitively for several years: the limit. The limit asks the question, "What does the function do as the independent variable gets closer and closer to a certain value?" In simpler terms, the limit is the "natural tendency" of a function. The limit is incredibly important due to its relationship to the derivative, the integral, and countless other key mathematical concepts. A strong understanding of the limit is essential to the field of mathematics.*

2.3.1 The Definition and Properties of Limits - Reading: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.3: Limits" Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.3: Limits" (PDF)

Instructions: Please click on the link above and read Section 2.3 (pages 36-45) in its entirety. Read this section carefully and pay close attention to the definition of the limit and the examples that follow. You should also closely examine the algebraic properties of limits as you

will need to take advantage of these in the exercises.

This reading should take you approximately two hours to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 2: Limits"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 2: Limits" (YouTube)

  Instructions: Please click on the link above and watch the entire video (52:47). Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.3: Problems 1-18"** Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.3: Problems 1-18" (PDF)

  Instructions: Please click on the link above and work through problems 1-18. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately two hours to complete.

- **Assignment: University of California at Davis: Duane Kouba's "Precise Limits of Functions as X Approaches a Constant: Problems 1-10"** Link: University of California at Davis: Duane Kouba's "Precise Limits of Functions as X Approaches a Constant: Problems 1-10" (HTML)

  Instructions: Please click on the link above and work through problems 1-10. When you are done, select the "click HERE" beneath each problem to check your solution.

  This assignment should take you approximately one hour to complete.

2.3.2 The Squeeze Theorem - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.3: A Hard Limit" Link: Whitman College:

David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.3: A Hard Limit" (PDF)

Instructions: Please click on the link above and read Section 4.3 (pages 75-77) in its entirety. The Squeeze Theorem is an important application of the limit and is useful in many limit computations.

This reading should take you approximately 30 minute to complete.

- **Web Media: PatrickJMT's "The Squeeze Theorem for Limits"** Link: PatrickJMT's "The Squeeze Theorem for Limits" (YouTube)

  Instructions: Please click on the link above and watch the entire video (7:13), which illustrates the Squeeze Theorem using specific examples.

  Viewing this video and taking notes should take you approximately 15 minutes to complete.

2.4 The Derivative Function - Reading: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.4: The Derivative Function" Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.4: The Derivative Function" (PDF)

Instructions: Please click on the link above and read Section 2.4 (pages 46-50) in its entirety. In this reading, you will see how limits are used to compute derivatives. A derivative is a description of how a function changes as its input varies. In the case of a straight line, this description is the same at every point, which is why we can describe the slope of an entire function when it is linear. You can also describe the slope of nonlinear functions. The slope, however, will not be constant; it will change as the independent variable changes.

This reading should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.4: Problems 1-5 and 8-11"** Link: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Exercises 2.4: Problems 1-5 and 8-11" (PDF)

  Instructions: Please click on the link above and work through problems 1-5 and 8-11. When you are done, check your answers against "Appendix A: Answers".

This assignment should take you approximately one hour to complete.

2.5 Adjectives for Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.5: Adjectives for Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 2: Instantaneous Rate of Change: The Derivative: "Section 2.5: Adjectives for Functions" (PDF)

Instructions: Please click on the link above and read Section 2.5 (pages 51-54) in its entirety. This reading covers the topics outlined in sub-subunits 2.5.1 through 2.5.3. The intuitive notion of a continuous function is made precise using limits. Additionally, you will be introduced to the Intermediate Value Theorem, which rigorously captures the intuitive behavior of continuous real-valued functions.

This reading should take you approximately one hour to complete.

2.5.1 Continuous Functions - Assignment: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: James Palermo and Molly M. Cow's "Extending Continuity at a Missing Point" Module and Gerardo Mendoza's "Discontinuities of Simple Piecewise Defined Functions" Module Link: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: James Palermo and Molly M. Cow's "Extending Continuity at a Missing Point" Module and Gerardo Mendoza's "Discontinuities of Simple Piecewise Defined Functions" Module (HTML)

Instructions: Please click on the link above select the "Index." Click on the number 26 next to "A Missing Value" to launch the first module and complete problems 15-26. Then return to the index and click on the number 27 next to "Discontinuities of simple piecewise defined functions" to launch the second module and complete problems 1-10. If at any time the problem set becomes too easy for you, feel free to move forward.

Completing this assignment should take you approximately two hours to complete.

2.5.2 Differentiable Functions - Assignment: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Dan Reich's "Differentiability of Simple Piecewise Functions" Module Link: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Dan Reich's "Differentiability of Simple Piecewise Functions" Module (HTML)

Instructions: Please click on the link above and select the "Index." Click on the number 39 next to "Differentiability" to launch the module and complete problems 1-10. If at any time the

problem set becomes too easy for you, feel free to move forward.

This assignment should take you approximately one hour to complete.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

2.5.3 The Intermediate Value Theorem - Web Media: PatrickJMT's "Intermediate Value Theorem" Link: PatrickJMT's "Intermediate Value Theorem" (YouTube)

Instructions: Click on the link above and watch the entire video (7:53) for an explanation of the Intermediate Value Theorem.

```
 Viewing this video and taking notes should take you approximately
15 minutes to complete.
```

Unit 3: Rules for Finding Derivatives *Computing a derivative requires computing a limit. Because limit computations can be rather involved, we like to minimize the amount of work we have to do in practice. In this unit, you will build your skill using some rules for differentiation which will speed up your calculations of derivatives. In particular, you will see how to differentiate the sum, difference, product, quotient, and composition of two or more functions. You will also learn rules for differentiating power functions, including polynomial and root functions.*

Unit 3 Time Advisory
This unit should take you approximately 12 hours to complete.

☐ Subunit 3.1: 1 hour

☐ Subunit 3.2: 1.5 hours

☐ Subunit 3.3: 2 hours

☐ Subunit 3.4: 2.5 hours

☐ Subunit 3.5: 5 hours ☐ Reading: 1 hour

☐ Lecture: 1 hour

☐ Assignment: 3 hours

Unit3 Learning Outcomes
Upon successful completion of this unit, the student will be able to:
- Use the power, product, quotient, and chain rules to calculate derivatives.

3.1 The Power Rule - Reading: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Section 3.1: The Power Rule" Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Section 3.1: The Power Rule" (PDF)

Instructions: Please click on the link above and read Section 3.1 (pages 55-57) in its entirety. This section will show you a simple rule for how to find the derivative of a power function without explicitly computing a limit.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Exercises 3.1: Problems 1-6"** Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Exercises 3.1: Problems 1-6" (PDF)

  Instructions: Please click on the above link and work through problems 1-6. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately 30 minutes to complete.

3.2 Linearity of the Derivative - Reading: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Section 3.2: Linearity of the Derivative" Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Section 3.2: Linearity of the Derivative" (PDF)

Instructions: Please click on the link above and read Section 3.2 (pages 58-59) in its entirety. In this reading, you will see how the derivative behaves with regards to addition and subtraction of functions and with scalar multiplication. That is, you will see that the derivative is a linear operation.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Exercises 3.2: Problems 1-9, 11, and 12"** Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Exercises 3.2: Problems 1-9, 11, and 12" (PDF)

  Instructions: Please click on the link above and work through problems 1-9, 11, and 12. When you are done, check your answers "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

3.3 The Product Rule - Reading: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Section 3.3: The Product Rule" Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Section 3.3: The Product Rule" (PDF)

Instructions: Please click on the link above and read Section 3.3 (pages 60-61) in its entirety. The naïve assumption is that the derivative of a product of two functions is the product of the derivatives of the two functions. This assumption is false. In this reading, you will see that the derivative of a product is slightly more complicated, but that it follows a definite rule called the product rule.

This reading should take you approximately 30 minutes to complete.

Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here. (PDF)

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Exercises 3.3: Problems 1-5"** Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Exercises 3.3: Problems 1-5" (PDF)

  Instructions: Please click on the link above and work through problems 1-5. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately 30 minutes to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

- **Assignment: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Dan Reich's "Derivatives – The Product Rule" Module** Link: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Dan Reich's "Derivatives – The Product Rule" Module (HTML)

  Instructions: Please click on the link above and select the "Index." Click on the number 44 next to "Product Rule" to launch the module and complete problems 1-10. If at any time the problem set becomes too easy for you, feel free to move forward.

  This assignment should take you approximately one hour to complete.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.4 The Quotient Rule - Assignment: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Dan Reich's "Derivatives - The Quotient Rule" Module Link: Temple University: Gerardo Mendoza and Dan Reich's Calculus on the Web: Calculus Book I: Dan Reich's "Derivatives - The Quotient Rule" Module (HTML)

```
 Instructions: Please click on the link above and select the
"Index." Click on the number 45 next to "The Quotient Rule" to
launch the module and complete problems 1-10. If at any time the
problem set becomes too easy for you, feel free to move forward.

 This assignment should take you approximately one hour to
complete.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

- **Reading: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Section 3.4: The Quotient Rule"** Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Section 3.4: The Quotient Rule" (PDF)

  Instructions: Please click on the link above and read Section 3.4 (pages 62-65) in its entirety. As with product of two functions, the derivative of a quotient of two functions is not simply the quotient of the two derivatives. This reading will introduce you to the quotient rule for differentiating a quotient of two functions. In particular, it will allow you to find the derivative of any rational function.

  This reading should take you approximately one hour to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Exercises 3.4: Problems 5, 6, 8, and 9"** Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Exercises 3.4: Problems 5, 6, 8, and 9" (PDF)

  Instructions: Please click on the link above and work through problems 5, 6, 8, and 9. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately 30 minutes to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

3.5 The Chain Rule - Reading: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Section 3.5: The Chain Rule" Link: Whitman College: David

Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Section 3.5: The Chain Rule" (PDF)

Instructions: Please click on the link above and read Section 3.5 (pages 65-69) in its entirety. The chain rule explains how the derivative applies to the composition of functions. Pay particular attention to Example 3.11, which works through a derivative computation where all of the differentiation rules of this unit are applied in finding the derivative of one function.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 4: Chain Rule"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 4: Chain Rule" (YouTube)

  Instructions: Please click on the link above and watch the entire video (46:03). Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 3: Rules for Finding Derivatives: "Exercises 3.5: Problems 1-20 and 36-39"** Link: Whitman College: David Guichard's *Calculus:* Chapter 3: Rules for Finding Derivatives: "Exercises 3.5: Problems 1-20 and 36-39" (PDF)

  Instructions: Please click on the link above and work through problems 1-20 and 36-39. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately three hours to complete.

Unit 4: Transcendental Functions *In this unit, you will investigate the derivatives of trigonometric, inverse trigonometric, exponential, and logarithmic functions. Along the way, you will develop a technique of differentiation called implicit differentiation. Aside from allowing you to compute derivatives of inverse function, implicit differentiation will also be important in studying related rates problems later on.*

Unit 4 Time Advisory
This unit should take you approximately 17 hours to complete.

- ☐ Subunit 4.1: 1.5 hours
- ☐ Subunit 4.2: 0.25 hourd
- ☐ Subunit 4.3: 1 hour
- ☐ Subunit 4.4: 1 hour
- ☐ Subunit 4.5: 1.25 hours
- ☐ Subunit 4.6: 0.5 hours
- ☐ Subunit 4.7: 3 hours
- ☐ Subunit 4.8: 3 hours
- ☐ Subunit 4.9: 2 hours
- ☐ Subunit 4.10: 2.5 hours
- ☐ Subunit 4.11: 1 hours

Unit4 Learning Outcomes
Upon successful completion of this unit, the student will be able to:
- Use implicit differentiation to find derivatives. - Find derivatives of inverse functions. - Find derivatives of trigonometric, exponential, logarithmic, and inverse trigonometric functions. - State and apply L'Hopital's Rule for indeterminate forms.

4.1 Trigonometric Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.1: Trigonometric Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.1: Trigonometric Functions" (PDF)

Instructions: Please click on the link above and read Section 4.1 (pages 71-74) in its entirety. This reading will review the definition of trigonometric functions.

This reading should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.1: Problems 1-4 and 11"** Link Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.1: Problems 1-4 and 11" (PDF)

  Instructions: Please click on the link above and work through problems 1-4 and 11. When you are done, check your answers against "Appendix A: Answers".

This assignment should take you approximately 30 minutes to complete.

4.2 The Derivative of Sine - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.2: The Derivative of sin x" Link: Whitman College: Professor David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.2: The Derivative of sin x" (PDF)

Instructions: Please click on the link above and read Section 4.2 (pages 74-75) in its entirety. This reading begins the computation of the derivative of the sine function. Two specific limits will need to be evaluated in order to complete this computation. These limits are addressed in the following section.

This reading should take you approximately 15 minutes to complete.

4.3 A Hard Limit - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.3: A Hard Limit" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.3: A Hard Limit" (PDF)

Instructions: Please click on the link above and read Section 4.3 (pages 75-77) in its entirety. You have read this section previously to become acquainted with the Squeeze Theorem. When you read the section the second time, pay particular attention to the geometric argument used to set up the application of the Squeeze Theorem.

```
 This reading should take you approximately 30 minutes to
complete.

 Terms of Use: This resource is licensed under a [Creative Commons
Attribution-NonCommercial-ShareAlike 3.0
License](http://creativecommons.org/licenses/by-nc-sa/3.0/).  This
text was originally written by Professor David Guichard.  It has
since been modified to include edited material from Neal Koblitz of
the University of Washington, H. Jerome Keisler of the University of
Wisconsin, Albert Schueller, Barry Balof, and Mike Wills.  You can
access the original version
[here](http://www.whitman.edu/mathematics/calculus/).
```

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.3: Problems 1-7"** Link: Professor David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.3: Problems 1-

7" (PDF)

Instructions: Please click on the link above and work through problems 1-7. When you are done, check your answers against "Appendix A: Answers".

This assignment should take you approximately 30 minutes to complete.

4.4 The Derivative of Sine, continued - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.4: The Derivative of sin x, continued" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.4: The Derivative of sin x, continued" (PDF)

Instructions: Please click on the link above and read Section 4.4 (pages 77-78) in its entirety. This reading completes the computation of the derivative of the sine function. Be sure to review all of the concepts involved in this computation.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.4: Problems 1-5"** Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.4, Problems 1-5" (PDF)

  Instructions: Please click on the link above and work through problems 1-5. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately 30 minutes to complete.

4.5 Derivatives of the Trigonometric Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.5: Derivatives of the Trigonometric Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.5: Derivatives of the Trigonometric Functions" (PDF)

Instructions: Please click on the link above and read Section 4.5 (pages 78 and 79) in its

entirety. Building on the work done to compute the derivative of the sine function and the rules of differentiation from previous readings, the derivatives of the remaining trigonometric functions are computed.

This reading should take you approximately 15 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.5: Problems 1-18"** Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.5: Problems 1-18" (PDF)

  Instructions: Please click on the link above and work through problems 1-18. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

4.6 Exponential and Logarithmic Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.6: Exponential and Logarithmic Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.6: Exponential and Logarithmic Functions" (PDF)

Instructions: Please click on the link above and read Section 4.6 (pages 80-81) in its entirety. This reading reviews the exponential and logarithmic functions, their properties, and their graphs.

This reading should take you approximately 30 minutes to complete.

4.7 Derivatives of the Exponential and Logarithmic Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.7: Derivatives of the Exponential and Logarithmic Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.7: Derivatives of the Exponential and Logarithmic Functions" (PDF)

Instructions: Please click on the link above and read Section 4.7 (pages 82-86) in its entirety. In this reading, the derivatives of the exponential and logarithmic functions are computed. Notice that the number e is defined in terms of a particular limit.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 6: Exponential and Log"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 6: Exponential and Log" (YouTube)

  Instructions: Please click on the link above and watch the entire video (47:57). Lecture notes are available here. Professor Jerison makes use of implicit differentiation at times during this lecture. You should take note of this and re-watch those portions of the video after completing subunit 4.8 below.

  Viewing this video and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.7: Problems 1-15 and 20"** Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.7: Problems 1-15 and 20" (PDF)

  Instructions: Please click on the link above and work through problems 1-15 and 20 for Exercise 4.7. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

4.8 Implicit Differentiation - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.8: Implicit Differentiation" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.8: Implicit Differentiation" (PDF)

Instructions: Please click on the link above and read Section 4.8 (pages 87-90) in its entirety. As a result of the chain rule, we have a method for computing derivatives of curves which are not explicitly described by a function. This method, called implicit differentiation, allows us to find tangent lines to such curves.

This reading should take you approximately one hour to complete.

ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 5: Implicit Differentiation"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 5: Implicit Differentiation" (YouTube)

  Instructions: Please click on the link above and watch the entire video (49:01). Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. It is attributed to David Jerison and MIT's OpenCourseWare. It may be viewed in its original form here.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.8: Problems 1-9 and 11-16"** Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.8: Problems 1-9 and 11-16" (PDF)

  Instructions: Please click on the link above and work through problems 1-9 and 11-16 for Exercises 4.8. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

4.9 Inverse Trigonometric Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.9: Inverse Trigonometric Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.9: Inverse Trigonometric Functions" (PDF)

Instructions: Please click on the link above and read Section 4.9 (pages 91-94) in its entirety. In this reading, implicit differentiation and the Pythagorean identity are used to compute the derivatives of inverse trigonometric functions. You should notice that the same techniques can be used to find derivatives of other inverse functions as well.

This reading should take you approximately one hour to complete.

Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.9: Problems 3-11"** Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.9: Problems 3-11" (PDF)

  Instructions: Please click on the link above and work through problems 3-11 for Exercises 4.9. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

4.10 Limits Revisited - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.10: Limits Revisited" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.10: Limits Revisited" (PDF)

Instructions: Please click on the link above and read Section 4.10 (pages 94-97) in its entirety. In this section, you will learn how derivatives relate back to limits. Limits of Indeterminate Forms (or limits of functions that, when evaluated, tend to 0/0 or ∞/∞) have previously been beyond our grasp. Using L'Hopital's Rule, you will find that these limits are attainable with derivatives.

This reading should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Exercises 4.10: Problems 1-10 and 21-24"** Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Exercises 4.10: Problems 1-10 and 21-24" (PDF)

  Instructions: Please click on the link above link and work through problems 1-10 and 21-24 for Exercise 4.10. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour and 30 minutes to complete.

4.11 Hyperbolic Functions - Reading: Whitman College: David Guichard's Calculus: Chapter 4: Transcendental Functions: "Section 4.11: Hyperbolic Functions" Link: Whitman College: David Guichard's *Calculus:* Chapter 4: Transcendental Functions: "Section 4.11:

Hyperbolic Functions" (PDF)

Instructions: Please click on the link above and read Section 4.11 (pages 99-102) in its entirety. In this reading, you are introduced to the hyperbolic trigonometric functions. These functions, which appear in many engineering and physics applications, are specific combinations of exponential functions which have properties similar to those that the ordinary trigonometric functions have.

This reading should take you approximately one hour to complete.

Unit 5: Curve Sketching *This section will ask you to apply a little critical thinking to the topics this course has covered thus far. To properly sketch a curve, you must analyze the function and its first and second derivatives in order to obtain information about how the function behaves, taking into account its intercepts, asymptotes (vertical and horizontal), maximum values, minimum values, points of inflection, and the respective intervals between each of these. After collecting this information, you will need to piece it all together in order to sketch an approximation of the original function.*

Unit 5 Time Advisory
This unit should take approximately 10.25 hours to complete.

- ☐ Subunit 5.1: 3 hours
- ☐ Subunit 5.2: 1.75 hours
- ☐ Subunit 5.3: 1.5 hours
- ☐ Subunit 5.4: 1.5 hours
- ☐ Subunit 5.5: 2.5 hours

Unit5 Learning Outcomes
Upon successful completion of this unit, the student will be able to:
- Define local and absolute extrema. - Use critical points to find local extrema. - Use the first and second derivative tests to find intervals of increase and decrease and to find information about concavity and inflection points. - Sketch functions using information from the first and second derivative tests.

5.1 Maxima and Minima - Reading: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Section 5.1: Maxima and Minima" Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Section 5.1: Maxima and Minima" (PDF)

Instructions: Please click on the link above and read Section 5.1 (pages 103-106) in its entirety. Fermat's Theorem indicates how derivatives can be used to find where a function reaches its highest or lowest points.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 10: Curve Sketching"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 10: Curve Sketching" (YouTube)

  Instructions: Please click on the link above and watch the video from the 30:00 minute mark to the end. Lecture notes are available here. The lecture will make use of the first and second derivative tests, which you will read about below.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Exercises 5.1: Problems 1-12 and 15"** Link: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Exercises 5.1: Problems 1-12 and 15" (PDF)

  Instructions: Please click on the link above and work through problems 1-12 and 15 for Exercises 5.1. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

5.2 The First Derivative Test - Reading: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Section 5.2: The First Derivative Test" Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Section 5.2: The First Derivative Test" (PDF)

Instructions: Please click on the link above and read Section 5.2 (page 107) in its entirety. In this reading, you will see how to use information about the derivative of a function to find local maxima and minima.

```
 This reading should take you approximately 15 minutes to
complete.

 Terms of Use: This resource is licensed under a [Creative Commons
Attribution-NonCommercial-ShareAlike 3.0
License](http://creativecommons.org/licenses/by-nc-sa/3.0/).  This
text was originally written by Professor David Guichard.  It has
```

since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version [here](http://www.whitman.edu/mathematics/calculus/).

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Exercises 5.2: Problems 1-15"** Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Exercises 5.2: Problems 1-15" (PDF)

  Instructions: Please click on the link above and work through problems 1-15 for Exercises 5.2. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour and 30 minutes to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

5.3 The Second Derivative Test - Reading: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Section 5.3: The Second Derivative Test" Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Section 5.3: The Second Derivative Test" (PDF)

Instructions: Please click on the link above and read Section 5.3 (pages 108-109) in its entirety. In this reading, you will see how to use information about the second derivative (that is, the derivative of the derivative) of a function to find local maxima and minima.

This reading should take you approximately 30 minutes to complete.

Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Exercises 5.3: Problems 1-10"** Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Exercises 5.3: Problems 1-10" (PDF)

  Instructions: Please click on the link above and work through problems 1-10 for Exercises 5.3. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

  Terms of Use: This resource is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License. This text was originally written by Professor David Guichard. It has since been modified to include edited material from Neal Koblitz of the University of Washington, H. Jerome Keisler of the University of Wisconsin, Albert Schueller, Barry Balof, and Mike Wills. You can access the original version here.

5.4 Concavity and Inflection Points - Reading: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Section 5.4: Concavity and Inflection Points" Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Section 5.4: Concavity and Inflection Points" (PDF)

Instructions: Please click on the link above and read Section 5.4 (pages 109-110) in its entirety. In this reading, you will see how the second derivative relates to the concavity of the graph of a function and use this information to find the points where the concavity changes, i.e. the inflection points of the graph.

This reading should take you approximately 30 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Exercises 5.4: Problems 1-9 and 19"** Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Exercises 5.4: Problems 1-9 and 19" (PDF)

  Instructions: Please click on the link above and work through problems 1-9 and 19 for Exercises 5.4. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

5.5 Asymptotes and Other Things to Look For - Reading: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Section 5.5: Asymptotes and Other Things to Look For" Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Section 5.5: Asymptotes and Other Things to Look For" (PDF)

Instructions: Please click on the link above and read Section 5.5 (pages 111-112) in its entirety. In this reading, you will see how limits can be used to find any asymptotes the graph of a function may have.

This reading should take you approximately 30 minutes to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 11: Max-min"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 11: Max-min" (YouTube)

Instructions: Please click on the link above and watch the video from the beginning to the 45:00 minute mark. Lecture notes are available here. The majority of the video lecture is about curve sketching, despite the title of the video.

Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 5: Curve Sketching: "Exercises 5.5: Problems 1-5 and 15-19"** Link: Whitman College: David Guichard's *Calculus:* Chapter 5: Curve Sketching: "Exercises 5.5: Problems 1-5 and 15-19" (PDF)

  Instructions: Please click on the link above and work through problems 1-5 and 15-19 for Exercises 5.5. When you are done, graph the curves using Wolfram Alpha to check your answers.

  This assignment should take you approximately one hour to complete.

Unit 6: Applications of the Derivative *With a sufficient amount of sophisticated machinery under your belt, you will now start to look at how differentiation can be used to solve problems in various applied settings. Optimization is an important notion in fields like biology, economics, and physics when we want to know when growth is maximized, for example. In addition to methods we use to solve problems directly, we can also use the derivative to find approximate solutions to problems. You will explore two such methods in this section: Newton's method and differentials.*

Unit 6 Time Advisory
This unit should take you approximately 13.25 hours to complete.

- ☐ Subunit 6.1: 5 hours
- ☐ Reading: 3 hours
- ☐ Lecture: 1 hour
- ☐ Assignments: 1 hour
- ☐ Subunit 6.2: 3 hours
- ☐ Subunit 6.3: 1.75 hours
- ☐ Subunit 6.4: 1.75 hours
- ☐ Subunit 6.5: 1.75 hours

Unit6 Learning Outcomes
Upon successful completion of this unit, the student will be able to:
- Solve problems involving rectilinear motion using derivatives. - Solve problems involving related rates. - Use the first and second derivative tests to solve optimization (maximum/minimum value) problems. - State and apply Rolle's Theorem and the Mean Value Theorem. - Explain the meaning of linear approximations and differentials with a sketch. - Use linear approximation to solve problems in applications. - State and apply L'Hopital's Rule for indeterminate forms. - Explain Newton's method using an illustration. - Execute several steps of Newton's method and use it to approximate solutions to a root-finding problem.

6.1 Optimization - Reading: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Section 6.1: Optimization" Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Section 6.1: Optimization" (PDF)

Instructions: Please click on the link above and read Section 6.1 (pages 115-124) in its entirety. An important application of the derivative is to find the global maximum and global minimum of a function. The Extreme Value Theorem indicates how to approach this problem. Pay particular attention to the summary at the end of the section.

This reading should take you approximately three hours to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 12: Related Rates"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 12: Related Rates" (YouTube)

  Instructions: Please click on the link above and watch the video from the beginning to the 45:00 minute mark. Lecture notes are available here. The majority of the video lecture is about optimization, despite the title of the video.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Exercises 6.1: Problems 5, 7, 9, 10, 14, 16, 22, 26, 28, and 33"** Link: Whitman College: Professor David Guichard's *Calculus:* "Chapter 6: Applications of the Derivative:" "Exercises 6.1, Problems 5, 7, 9, 10, 14, 16, 22, 26, 28, 33" (PDF)

  Instructions: Please click on the link above link and work through problems 5, 7, 9, 10, 14, 16, 22, 26, 28, and 33 for Exercises 6.1. When you are done, to check your answers against "Appendix A: Answers".

This assignment should take you approximately one hour to complete.

6.2 Related Rates *Note: You now know how to take the derivative with respect to the independent variable. In other words, you know how to determine a function's rate of change when given the input's rate of change. But what if the independent variable was itself a function? What if, for example, the input was a function of time? How do we identify how the function changes as time changes? This subunit will explore the answers to these questions.*

- **Reading: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Section 6.2: Related Rates"** Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Section 6.2: Related Rates" (PDF)

  Instructions: Please click on the link above and read Section 6.2 (pages 127-132) in its entirety. Another application of the chain rule, related rates problems apply to situations where multiple dependent variables are changing with respect to the same independent variable. Make note of the summary in the middle of page 128.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 13: Newton's Method"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 13: Newton's Method" (YouTube)

  Instructions: Please click on the link above and watch the video from the beginning to the 40:30 minute mark. Lecture notes are available here. The majority of the video lecture is about related rates, despite the title of the video.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Exercises 6.2: Problems 1, 3, 5, 11, 14, 16, 19-21, and 25"** Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Exercises 6.2: Problems 1, 3, 5, 11, 14, 16, 19-21, and 25" (PDF)

  Solutions: *Ibid:* "Appendix A: Answers" (PDF)

  Instructions: Please click on the above link, and work through problems 1, 3, 5, 11, 14, 16, 19-21, and 25 for Exercises 6.2. When you are done, check your answers against "Appendix A:

Answers".

This assignment should take you approximately one hour to complete.

6.3 Newton's Method *Note: Newton's Method is a process by which we estimate the roots of a real-valued function. You may remember the bisection method, whereby we find a root by creating smaller and smaller intervals. Newton's Method uses the derivative in order to account for both the speed at which the function changes and its actual position. This creates an algorithm that can help us identify the location of roots even more quickly.*

Newton's Method requires that you start "sufficiently close" (a somewhat arbitrary specification that varies from problem to problem) to the actual root in order to estimate it with accuracy. If you start too far from the root, an algorithm can be led awry in certain situations.

- **Reading: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Section 6.3: Newton's Method"** Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Section 6.3: Newton's Method" (PDF)

  Instructions: Please click on the link above and read Section 6.3 (pages 135-138) in its entirety. In this section, you will be introduced to a numerical approximation technique called Newton's Method. This method is useful for finding approximate solutions to equations which cannot be solved exactly.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 14: The Mean Value Theorem"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 14: The Mean Value Theorem" (YouTube)

  Instructions: Please click on the link above watch the video from the beginning to the 15:10 minute mark. Lecture notes are available here. This portion of the video is about Newton's Method, despite the title of the video.

  Viewing this lecture and taking notes should take you approximately 15-20 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Exercises 6.3: Problems 1-4"** Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Exercises 6.3: Problems 1-4" (PDF)

  Instructions: Please click on the link above link and work through problems 1-4 for Exercises 6.3. When you are done, check your answers "Appendix A: Answers".

This assignment should take you approximately 30 minutes to complete.

6.4 Linear Approximations *Note: In this subunit, you will learn how to estimate future data points based on what you know about a previous data point and how it changed at that particular moment. This concept is extremely useful in the field of economics.*

- **Reading: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Section 6.4: Linear Approximations"** Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Section 6.4: Linear Approximations" (PDF)

  Instructions: Please click on the link above and read Section 6.4 (pages 139-140) in its entirety. In this reading, you will see how tangent lines can be used to locally approximate functions.

  This reading should take you approximately 30 minutes to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 9: Linear and Quadratic Approximations"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 9: Linear and Quadratic Approximations" (YouTube)

  Instructions: Please click on the link above and watch the video up to the 39:00 minute mark. At the 39:00 mark Professor Jerison begins to discuss quadratic approximations to functions, which are in a certain sense one step beyond linear approximations. If you are interested, please continue viewing the lecture to the end. Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately 45 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Exercises 6.4: Problems 1-4"** Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Exercises 6.4: Problems 1-4" (PDF)

  Instructions: Please click on the link above link and work through problems 1-4 for Exercises 6.4. When you are done, check your answers against "Appendix A: Answers". Please note that the correct answer for 6.4.4 is actually (highlight to see the correct answer).

  This assignment should take you approximately 30 minutes to complete.

6.5 The Mean Value Theorem - Reading: Whitman College: David Guichard's Calculus: Chapter 6: Applications of the Derivative: "Section 6.5: The Mean Value Theorem" Link: Whitman College: David Guichard's *Calculus:* Chapter 6: Applications of the Derivative: "Section 6.5: The Mean Value Theorem" (PDF)

Instructions: Please click on the link above and read Section 6.5 (pages 141-144) in its entirety. The Mean Value Theorem is an important application of the derivative which is used most often in developing further mathematical theories. A special case of the Mean Value Theorem, called Rolle's Theorem, leads to a characterization of antiderivatives.

This reading should take you approximately 30 minutes to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 14: The Mean Value Theorem"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 14: The Mean Value Theorem" (YouTube)

  Instructions: Please click on the link above and watch the video from the 15:10 minute mark to the end. Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately 45 minutes to complete.

- **Assignment: Whitman College: Professor David Guichard's Calculus: "Chapter 6: Applications of the Derivative:" "Exercises 6.5, Problems 1, 2, 6-9"** Link: Whitman College: Professor David Guichard's *Calculus:* "Chapter 6: Applications of the Derivative": "Exercises 6.5, Problems 1, 2, 6-9" (PDF)

Solutions: *Ibid:* "Appendix A: Answers" (PDF)

Instructions: Please click on the above link, and work through problems 1, 2, and 6-9 for Exercise 6.5. When you are done, click the second link to check your answers.

This assignment should take approximately 30 minutes to complete.

Unit 7: Integration *In the last unit of this course, you will learn about "integral calculus," a subfield of calculus that studies the area formed under the curve of a function. Although its relationship with the derivative is not necessarily intuitive, integral calculus is closely linked to the derivative, which you will revisit in this unit.*

Unit 7 Time Advisory
This unit should take you approximately 15.25 hours to complete.

☐ Subunit 7.1: 3 hours

☐ Subunit 7.2: 5.5 hours ☐ Reading: 1.5 hours

☐ Lecture: 2 hours

☐ Assignment: 2 hours

☐ Subunit 7.3: 2.5 hours

☐ Subunit 7.4: 4.25 hours ☐ Reading: 1.5 hours

☐ Lecture: 0.75 hours

☐ Assignment: 2 hours

Unit7 Learning Outcomes
Upon successful completion of this unit, the student will be able to:
- Define antiderivatives and the indefinite integral. - State the properties of the indefinite integral. - Relate the definite integral to the initial value problem and the area problem. - Set up and calculate a Riemann sum. - State the Fundamental Theorem of Calculus and use it to calculate definite integrals. - State and apply basic properties of the definite integral. - Use substitution to compute definite integrals.

7.1 Motivation - Reading: Whitman College: David Guichard's Calculus: Chapter 7: Integration: "Section 7.1: Two Examples" Link: Whitman College: David Guichard's *Calculus:* Chapter 7: Integration: "Section 7.1: Two Examples" (PDF)

Instructions: Please click on the link above and read Section 7.1 (pages 145-149) in its entirety. This reading introduces the integral through two examples. The first example addresses the question of how to determine the distance traveled based only on information about

velocity. The second example addresses the question of how to determine the area under the graph of a function. Surprisingly, these two questions are closely related to each other and to the derivative.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 18: Definite Integrals"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 18: Definite Integrals" (YouTube)

  Instructions: Please click on the link above and watch the entire video (47:14). Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 7: Integration: "Exercises 7.1: Problems 1-8"** Link: Whitman College: David Guichard's *Calculus:* Chapter 7: Integration: "Exercises 7.1: Problems 1-8" (PDF)

  Instructions: Please click on the link above and work through problems 1-8 for Exercises 7.1. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately one hour to complete.

7.2 The Fundamental Theorem of Calculus *Note: The Fundamental Theorem of Calculus is the apex of our course. It explains the relationship between the derivative and the integral, tying the two major facets of this course together. In the previous section, you learned the definition of the definite integral as a limit of a Riemann Sum. The computations were long and involved. In this subunit, you will learn about the Fundamental Theorem of Calculus, which makes the computation of definite integrals significantly easier.*

- **Reading: Whitman College: David Guichard's Calculus: Chapter 7: Integration: "Section 7.2: The Fundamental Theorem of Calculus"** Link: Whitman College: David Guichard's *Calculus:* Chapter 7: Integration: "Section 7.2: The Fundamental Theorem of Calculus" (PDF)

  Instructions: Please click on the link above and read Section 7.2 (pages 149-155) in its

entirety. Pay close attention to the treatment of Riemann sums, which lead to the definite integral. The Fundamental Theorem of Calculus explicitly describes the relationship between integrals and derivatives.

This reading should take you approximately one hour and 30 minutes to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus*: "Lecture 19: The First Fundamental Theorem"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 19: The First Fundamental Theorem" (YouTube)

  Instructions: Watch this video lecture. Lecture notes are available here.

Watching this lecture and taking notes should take you approximately one hour.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 20: The Second Fundamental Theorem"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 20: The Second Fundamental Theorem" (YouTube)

  Instructions: Please click on the link above and watch the entire video (49:30). Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately one hour to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 7: Integration: "Exercises 7.2: Problems 7-22"** Link: Whitman College: David Guichard's *Calculus:* Chapter 7: Integration: "Exercises 7.2: Problems 7-22" (PDF)

  Instructions: Please click on the link above and work through problems 7-22 for Exercises 7.2. When you are done, check your answers against "Appendix A: Answers".

This assignment should take you approximately two hours to complete.

7.3 Some Properties of Integrals - Reading: Whitman College: David Guichard's Calculus: Chapter 7: Integration: "Section 7.3: Some Properties of Integrals" Link: Whitman College: David Guichard's *Calculus:* Chapter 7: Integration: "Section 7.3: Some Properties of Integrals" (PDF)

Instructions: Please click on the link above and read Section 7.3 (pages 156-160) in its entirety. In particular, note that the definite integral enjoys the same linearity properties that the derivative does, in addition to some others. In its application to velocity functions, pay particular attention to the distinction between distance traveled and net distance traveled.

This reading should take you approximately one hour to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 15: Antiderivatives"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 15: Antiderivatives" (YouTube)

  Instructions: Please click on the link above and watch the video from the beginning to the 30:00 minute mark. Lecture notes are available here.

  Viewing this lecture and taking notes should take you approximately 45 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 7: Integration: "Exercises 7.3: Problems 1-6"** Link: Whitman College: David Guichard's *Calculus:* Chapter 7: Integration: "Exercises 7.3: Problems 1-6" (PDF)

  Instructions: Please click on the link above link and work through problems 1-6 for Exercises 7.3. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately 45 minutes to complete.

7.4 Integration by Substitution - Reading: Whitman College: David Guichard's Calculus: Chapter 8: Techniques of Integration: "Section 8.1: Substitution" Link: Whitman College: David Guichard's *Calculus:* Chapter 8: Techniques of Integration: "Section 8.1: Substitution" (PDF)

Instructions: Please click on the link above and read Section 8.1 (pages 161-166) in its entirety. This section explains the process of taking the integral of slightly more complicated

functions. We do this by implementing a "change of variables," or rewriting a complicated integral in terms of elementary functions that we already know how to integrate. Simply put, integration by substitution is merely the act of taking the chain rule in reverse.

This reading should take approximately one hour and 30 minutes to complete.

- **Lecture: Massachusetts Institute of Technology: David Jerison's Single Variable Calculus: "Lecture 15: Antiderivatives"** Link: Massachusetts Institute of Technology: David Jerison's *Single Variable Calculus:* "Lecture 15: Antiderivatives" (YouTube)

  Instructions: Please click on the link above and watch the video from the 30:00 minute mark to the end. Lecture notes are available here.

Viewing this lecture and taking notes should take you approximately 45 minutes to complete.

- **Assignment: Whitman College: David Guichard's Calculus: Chapter 8: Techniques of Integration: "Exercises 8.1: Problems 5-19"** Link: Whitman College: David Guichard's *Calculus:* Chapter 8: Techniques of Integration: "Exercises 8.1: Problems 5-19" (PDF)

  Instructions: Please click on the link above link and work through problems 5-19 for Exercises 8.1. When you are done, check your answers against "Appendix A: Answers".

  This assignment should take you approximately two hours to complete.

Unit 1: The Integral *We will begin by quickly reviewing the basics of integration, so integration is fresh in your mind before we extend its applications. Having completed MA101, you should be familiar with this material. We will then take a look at how integration applies to concepts like motion. Finally, we will discuss how logarithmic and exponential functions are integrated.*

Unit 1 Time Advisory
This unit should take you approximately 20.25 hours to complete.

☐ Subunit 1.1: 14 hours

☐ Sub-subunit 1.1.1: 2 hours

- ☐ Sub-subunit 1.1.2: 2 hours
- ☐ Sub-subunit 1.1.3: 1.5 hours
- ☐ Sub-subunit 1.1.4: 4 hours
- ☐ Sub-subunit 1.1.5: 2.5 hours
- ☐ Sub-subunit 1.1.6: 2 hours
- ☐ Subunit 1.2: 6.25 hours
- ☐ Sub-subunit 1.2.1: 2.5 hours
- ☐ Sub-subunit 1.2.2: 1.5 hours
- ☐ Sub-subunit 1.2.3: 2.25 hours

Unit1 Learning Outcomes
Upon successful completion of this unit, the student will be able to: - Define and describe the indefinite integral. - Compute elementary definite and indefinite integrals. - Explain the relationship between the area problem and the indefinite integral. - Use the midpoint rule to approximate the area under a curve. - State the fundamental theorem of calculus. - Use change of variables to compute more complicated integrals. - Integrate transcendental, logarithmic, and hyperbolic functions.

1.1 Review of Integration 1.1.1 The Indefinite Integral - Reading: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan's and Donald J. Lewis's Calculus and Linear Algebra Vol.1: "4-1 Introduction" and "4-2 The Indefinite Integral" Link: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan and Donald J. Lewis's *Calculus and Linear Algebra Vol. 1*: "4-1 Introduction" (HTML) and "4-2 The Indefinite Integral" (HTML)

```
 Instructions: Please click on the links above, and read Sections
4-1 and 4-2 in their entirety.  Note that for Section 4-2, you will
need to click on the "next" link at the bottom of each page to
continue the reading.

 How does one "undo" differentiation?  There are a few
considerations.  We need to loosely define "elementary function" to
mean a function put together from rational and trigonometric
functions, exponentials, radicals, and so forth.  Although the
derivative of an elementary function always is elementary, there are
elementary functions with no elementary antiderivative.  Since the
derivative of a constant is 0, two functions that differ only by an
added constant term will have the same derivative, so
antiderivatives are never unique.  However, these limitations do not
forbid us from developing a strong theory of antidifferentiation,
introduced in the readings linked above.

 Studying these readings should take approximately 30 minutes.
```

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Reading: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan and Donald J. Lewis's Calculus and Linear Algebra Vol. 1: "4- 5 Basic Properties of the Indefinite Integral" and "4-6 Applications of Rules of Integration"** Link: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan's and Donald J. Lewis's *Calculus and Linear Algebra Vol. 1*: "4-5 Basic Properties of the Indefinite Integral" (HTML) and "4-6 Applications of Rules of Integration" (HTML)

Instructions: Please click on the links above, and read Sections 4-5 and 4-6 in their entirety. Use the "previous" and "next" links at the bottom of the page to navigate through each reading. These readings discuss the "nuts and bolts" of finding antiderivatives.

Studying these reading should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Matthias Beck and Molly M. Cow's "Indefinite Integrals"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Matthias Beck and Molly M. Cow's "Indefinite Integrals" (HTML)

Instructions: Click on the above link. Then, click on the "Index" button. Scroll down to "1. Integration," and click button 104 (Indefinite Integrals). Do problems 1-10. Use the buttons in the module to check your answer or to move on to subsequent problems. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.1.2 The Area Problem and the Definite Integral - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 4: Integration: "Section 4.1: The Definite Integral" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 4: Integration: "Section 4.1: The Definite Integral" (PDF)

Instructions: Please click on the link above, and read Section 4.1 in its entirety (pages 175 through 185).

This is a slightly different presentation of definite integrals and area from the one chosen for MA101. The author approaches limits using the idea of rigorously defined *infinitesimals*, which you may think of as infinitely small numbers. These ultimately give a fairly intuitive theory, but you will need some vocabulary, because it is not the typical presentation. An *infinitesimal* is a number strictly between –a and a, for every positive real a. The only real infinitesimal is 0; more appear when we use the *hyperreals*, which are a collection of numbers extending the reals. There are non-real hyperreals between any two reals: for example, if ε is a positive infinitesimal, 5+ε is a non-real hyperreal between 5 and 6 (or even 5 and 5.00000001). The *standard part* of a hyperreal is obtained by "rounding to the nearest real," so the standard parts of 5+ε and 5–ε are both 5, and the standard part of ε by itself is 0. Division

by a nonzero infinitesimal gives an infinite hyperreal, a hyperreal that is strictly larger than any real number. The author is careful in the text to confirm the calculations give *finite* hyperreals. Infinite hyperreals have no standard part. The sum of two infinitesimals is always infinitesimal, as is the product of an infinitesimal and any finite hyperreal number.

The *Transfer Principle* states that any mathematical statement true of a function on the reals is also true of that function's natural extension to the hyperreals. For example, the fact that (x + y)² = x² + 2xy + y² holds for all real x and y means it also holds for all hyperreal x and y. The proof of this principle is not important for our purposes, but the principle itself is vital. Finally, a *hyperinteger* is the integer analog to a hyperreal. However, the only finite hyperintegers are the integers, so you may think of the hyperintegers as an extension of the integers into infinity.

Studying this reading should take approximately 1 hour.

Terms of Use: The article above is released under a [Creative Commons Attribution-NonCommercial-ShareAlike License 3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML). It is attributed to H. Jerome Kiesler and the original version can be found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).

- **Lecture: MIT: David Jerison's "Lecture 18: Definite Integrals" and University of Houston: Selwyn Hollis's "Video Calculus: The Integral" Lecture** Link: MIT: David Jerison's "Lecture 18: Definite Integrals" (YouTube)

Also Available in:
iTunes U

and University of Houston: Selwyn Hollis's "Video Calculus: The Integral" (QuickTime)

Instructions: Please watch the first video lecture in its entirety (47:14 minutes). Note that lecture notes are available in PDF; the link is on the same page as the lecture.

If you desire a shorter presentation, choose the second video; click on the second link; then scroll down to Video 22: "The Integral." Professor Jerison discusses the area problem and the cumulative sum problem and uses them to define the definite integral.

Studying one of these lectures should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.1.3 Approximating Integrals - Lecture: University of Houston: Selwyn Hollis's "Video Calculus: The Area under a Curve" Link: University of Houston: Selwyn Hollis's "Video Calculus: The Area under a Curve" (QuickTime)

Instructions: Please click on the link; then scroll down to Video 21: "The Area under a Curve." Please watch the entire lecture. We will return to the problem of approximating definite integrals

numerically in a later unit using more complicated methods.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: James Palermo and Matthias Beck's "Midpoint Rule"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: James Palermo and Matthias Beck's "Midpoint Rule" (HTML)

Instructions: Click on the above link. Then, click on the "Index" button. Scroll down to "1. Integration," and click button 111 (Midpoint Rule). Do problems 1-5. If you do not fully understand the midpoint rule, click on the "Help" button for a quick refresher. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.1.4 The Fundamental Theorem of Calculus *The Fundamental Theorem of Calculus is a two part statement relating definite integrals to derivatives and antiderivatives. First, it says that integrating f(t) from a to x and then differentiating results in f(x). Second, it says that definite integrals may be calculated via antiderivatives. It is important, because first, it says the relationship between integration and differentiation is what it ought to be, and second, that we can evaluate definite integrals without always resorting to limits of Riemann sums.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 4: Integration: "Section 4.2: The Fundamental Theorem of Calculus"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 4: Integration: "Section 4.2: The Fundamental Theorem of Calculus" (PDF)

Instructions: Please click on the link above, and read Section 4.2 in its entirety (pages 186 through 197). This is a recap of the fundamental theorem of calculus, which relates the area problem and the definite integral to antiderivatives.

Studying this reading should take approximately 1 hour.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: MIT: David Jerison's "Lecture 19: First Fundamental Theorem of Calculus"** Link: MIT: David Jerison's "Lecture 19: First Fundamental Theorem of Calculus" (YouTube)

Instructions: Please watch this video lecture in its entirety. This lecture will cover sub-subunits 1.1.3-1.1.5. Professor Jerison states the first fundamental theorem of calculus and uses it to calculate some integrals. He then discusses some properties of the definite integral and the method of substitution for computing integrals.

Viewing this lecture and pausing to take notes should take approximately 1 hour.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: The Fundamental Theorem of Calculus"** Link: University of Houston: Selwyn Hollis's "Video Calculus: The Fundamental Theorem of Calculus" (QuickTime)

Instructions: This video lecture is optional. Please click on the link; then scroll down to Video 23: "The Fundamental Theorem of Calculus." This video explains the definition of a function via integration and the differentiation of such functions.

Viewing this lecture and pausing to take notes should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Aaron Robertson's "Differentiation and the Fundamental Theorem"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Aaron Robertson's "Differentiation and the Fundamental Theorem" (HTML)

Instructions: Click on the above link. Then, click on the "Index" button. Scroll down to "1. Integration," and click button 114 (Differentiation and the Fundamental Theorem). Do problems 4-20. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour and 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.1.5 Elementary Integrals - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 4: Integration: "Section 4.3: The Indefinite Integral" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 4: Integration: "Section 4.3: The Indefinite Integral" (PDF)

```
 Instructions:  Please click on the link above, and read Section 4.3
in its entirety (pages 198 through 207).  In this text, the author
chooses to present the indefinite integral after the definite
integral; however, this should not interfere with your understanding
of the chapter.  The section is extremely well-written; pay close
attention to the discussion of antiderivatives, theorem 3, the rules
of integration, and example 9.

 Studying this reading should take approximately 1 hour.

 Terms of Use: The text above is released under a [Creative Commons
Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Antidifferentiation and Indefinite Integrals"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Antidifferentiation and Indefinite Integrals" (QuickTime)

Instructions: Please click on the link scroll down to Video 24: "Antidifferentiation and Indefinite Integrals," and view the entire lecture.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Daniel Russo's "Definite Integrals"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Daniel Russo's "Definite Integrals" (HTML)

Instructions: Click on the above link. Then, click on the "Index" button. Scroll down to "1. Integration," and click button 108 (Definite Integrals). Do problems 5-15. These should be very easy. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.1.6 Integration by Substitution (Change of Variables) *The derivative of f(g(x)), by the chain rule, is f'(g(x))*g'(x). If presented with the latter expression as our integrand, we should get f(g(x)) back as the integral. However, it can be difficult to determine what f and g are at a glance. Substitution "cleans up" the integrand by hiding g(x) inside a new variable u, and combining g'(x) and dx into du. It may also be used to turn a root into a power: let u equal the radical, solve for x, then differentiate, and plug the result in for dx. We will later see other uses of substitution, but all are designed to rewrite the problem into a form where the method of integration is clear.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 4: Integration: "Section 4.4: Integration by Change of Variables"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 4: Integration: "Section 4.4: Integration by Change of Variables" (PDF)

Instructions: Please click on the link above, and read Section 4.4 in its entirety (pages 209 through 215).

Studying this reading should take approximately 30 minutes.

Terms of Use: The above book is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Change of Variables (Substitution)"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Change of Variables (Substitution)" (QuickTime)

Instructions: Please click on the link, scroll down to Video 25: "Change of Variables (Substitution)," and view the entire lecture.

Viewing this lecture and pausing to take notes should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Dan Reich's "Substitution Methods"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Dan Reich's "Substitution Methods" (HTML)

Instructions: Click on the above link. Then, click on the "Index" button. Scroll down to "1. Integration," and click button 109 (Substitution Methods). Do at least problems 1-10. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.2 Integration of Transcendental Functions *A transcendental number is a number that is not the root of any integer polynomial. A transcendental function, similarly, is a function that cannot be written using roots and the arithmetic found in polynomials. We address exponential, logarithmic, and hyperbolic functions here, having covered the integration and differentiation of trigonometric functions previously.*

1.2.1 Exponential Functions - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 8: Exponential and Logarithmic Functions: "Section 8.3: Derivatives of Exponential Functions and the Number e" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.3: Derivatives of Exponential Functions and the Number e" (PDF)

```
 Instructions: Please click on the link above, and read Section 8.3
in its entirety (pages 441 through 447).  This chapter recaps the
definition of the number e and the exponential function and its
behavior under differentiation and integration.

 Studying this reading should take approximately 45 minutes.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: The Natural Logarithmic Function" and "The Exponential Function"** Link: University of Houston: Selwyn Hollis's "Video Calculus: The Natural Logarithmic Function" (QuickTime) and "The Exponential Function" (QuickTime)

Instructions: These lectures will cover sub-subunits 1.2.1 and 1.2.2. Please watch these two video lectures *AFTER* doing the readings for 1.2.1 and 1.2.2. Click on the first link, scroll down to Video 31: "The Natural Logarithmic Function," and watch the presentation through the 5th slide (marked 5 of 8). Next, click on the second link above, and scroll down to Video 32: "The Exponential Function." Choose the format that is most appropriate for your Internet connection, and listen to the entire 21 minute video lecture.

The first short video gives one definition of the natural logarithm and derives all the properties of the natural log from that definition. It does a number of examples of limits, curve sketching, differentiation, and integration using the natural log. We will return to this video later to watch the last three slides. The second video explains the number e, the exponential function and its

derivative and antiderivative, curve sketching using the exponential function, and how to perform similar operations on power functions with other bases using the change of base formula.

Viewing these lectures and note-taking should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: University of California, Davis: Duane Kouba's "The Integration of Exponential Functions: Problems 1-12"** Link: University of California, Davis: Duane Kouba's "The Integration of Exponential Functions: Problems 1-12" (HTML)

Instructions: Click on the link above and work through all of the assigned problems. When you are done, check your solutions with the answers provided.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.2.2 Natural Logarithmic Functions - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 8: Exponential and Logarithmic Functions: "Section 8.5: Natural Logarithms" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.5: Natural Logarithms" (PDF)

```
 Instructions: Please click on the link above, and read Section 8.5
in its entirety (pages 454 through 459).  This chapter reintroduces
the natural logarithm (the logarithm with base e) and discusses its
derivative and antiderivative.  Recall that you can use these
properties of the natural log to extrapolate the same properties for
logarithms with arbitrary bases by using the change of base
formula.

 Studying this reading should take approximately 30 minutes.

 Terms of Use: The book above is released under a [Creative Commons
Attribution-NonCommercial-Share-Alike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Matthias Beck's "Logarithm, Definite Integrals"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Matthias Beck's "Logarithms, Definite Integrals" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "3. Transcendental Functions," and click button 137 (Logarithm, Definite Integrals). Do problems 1-10. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

1.2.3 Hyperbolic Functions - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 8: Exponential and Logarithmic Functions: "Section 8.4: Some Uses of Exponential Functions" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.4: Some Uses of Exponential Functions" (PDF)

```
 Instructions: Please click on the link above and read Section 8.4
in its entirety (pages 449 through 453).  In this chapter, you will
learn the definitions of the hyperbolic trig functions and how to
differentiate and integrate them.  The chapter also introduces the
concept of capital accumulation.

 Studying this reading should take approximately 15-20 minutes.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Lecture: YouTube: Gaussian Technologies: GaussianMath.com's "Hyperbolic Functions" and "Hyperbolic Functions – Derivatives"** Link: YouTube: Gaussian Technologies: GaussianMath.com's "Hyperbolic Functions" (YouTube) and "Hyperbolic Functions – Derivatives" (YouTube)

  Instructions: Click on the links above and watch the videos. The creator of the video pronounces "sinh" as "chingk." The more usual pronunciation is "sinch."

  Viewing these lectures and pausing to take notes should take approximately 30 minutes.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Hyperbolic Functions"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Hyperbolic Functions" (PDF)

  Also Available in:

  HTML

  Instructions: Please click on the link above, and work through each of the sixteen examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

  Completing this assessment should take approximately 1 hour and 30 minutes.

Unit 2: Applications of Integration *In this unit, we will take a first look at how integration can and has been used to solve various types of problems. Now that you have conceptualized the relationship between integration and areas and distances, you are ready to take a closer look at various applications; these range from basic geometric identities to more advanced situations in Physics and Engineering.*

Unit 2 Time Advisory
This unit should take you 19.75 hours to complete.

- ☐ Subunit 2.1: 2.75 hours
- ☐ Subunit 2.2: 1 hour
- ☐ Subunit 2.3: 4.5 hours
- ☐ Sub-subunit 2.3.1: 3 hours
- ☐ Sub-subunit 2.3.2: 1.5 hours
- ☐ Subunit 2.4: 2.5 hours
- ☐ Subunit 2.5: 1.5 hours
- ☐ Subunit 2.6: 2 hours
- ☐ Subunit 2.7: 5.5 hours
- ☐ Sub-subunit 2.7.1: 1.5 hours
- ☐ Sub-subunit 2.7.2: 1.5 hours
- ☐ Sub-subunit 2.7.3: 0.5 hours
- ☐ Sub-subunit 2.7.4: 2 hours

Unit2 Learning Outcomes
Upon successful completion of this unit, the student will be able to: - Find the area between two curves. - Find the volumes of solids using ideas from geometry. - Find the volumes of solids of revolution using disks and washers. - Find the volumes of solids of revolution using shells. - Write and interpret a parameterization for a curve. - Find the length of a curve. - Find the surface area of a solid of revolution. - Compute the average value of a function. - Use integrals to compute the displacement and the total distance traveled. - Use integrals to compute moments and centers of mass. - Use integrals to compute work.

2.1 The Area between Curves *Suppose you want to find the area between two concentric circles. How would you do this? Logic dictates that you subtract the area of the smaller circle from that of the larger circle. As this subunit will demonstrate, this method also works when you are trying to determine the area between curves.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 4: Integration: "Section 4.5: Areas between Two Curves"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 4: Integration: "Section 4.5: Areas between Two Curves" (PDF)

Instructions: Please click on the link above, and read Section 4.5 in its entirety (pages 218 through 222).

Studying this reading should take approximately 30 minutes.

- **Lecture: YouTube: MIT: David Jerison's "Lecture 21: Applications to Logarithms and Geometry"** Link: YouTube: MIT: David Jerison's "Lecture 21: Applications of Logarithms and Geometry" (YouTube)

  Also Available in:
  iTunes U

  Instructions: Please watch the segment of this video lecture from time 21:30 minutes through the end. Note that lecture notes are available in PDF; the link is on the same page as the lecture. In this lecture, Dr. Jerison will explain how to calculate the area between two curves.

  Viewing this lecture and pausing to take notes should take approximately 45 minutes.

  Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Aaron Robertson's "Area between Curves I" and "Area between Curves II"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Aaron Robertson's "Area between Curves I" (HTML) and "Area between Curves II" (HTML)

  Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Applications of Integration," and click button 115 (Area between Curves I). Do problems 6-13. Next, choose button 116 (Area between Curves II), and do problems 4-10. If at any time a problem set seems too easy for you, feel free to move on.

  Completing these assessments should take approximately 1 hour.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Indiana University Southeast: Margaret Ehringe's "Practice on Area between Two Curves"** Link: Indiana University Southeast: Margaret Ehringe's "Section 5.3 Area between Two Curves" (HTML)

  Instructions: Click on the link above, and do problems 1-3 and 6-9. When you have finished, scroll down the page to check your answers.

  The point of this third assessment is for you to practice setting up and completing these problems without the graphical aids provided by the Temple University media; you will have to graph these curves for yourself in order to begin the problems.

  Completing this assessment should take approximately 30 minutes.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

2.2 Volumes of Solids *We often take basic geometric formulas for granted. (Have you ever asked yourself why the volume of a right cylinder is $V=\pi hr^2$?) In this subunit, we will explore how some of these formulas were developed. The key lies in viewing solids as functions that revolve around certain lines. Consider, for example, a constant, horizontal line, and then imagine that line revolving around the x-axis (or any parallel line). The resulting shape is a right cylinder. We can find the volume of this figure by looking at infinitesimally thin "slices" and adding them all together. This concept enables us to calculate the volume of some extremely*

complex figures. In this subunit, we will learn how to do this in general; in the next, we will now take a look at two conventional methods for doing so when the figure has rotational symmetry.

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Volumes I"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Volumes I" (QuickTime)

  Instructions: Please click on the link, scroll down to Video 27: "Volumes I," and view the entire video. This video explains how to use integral calculus to calculate the volumes of general solids.

  Viewing this lecture and pausing to take notes should take approximately 30 minutes.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Finding Volumes by Slicing"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Finding Volumes by Slicing" (PDF)

  Also Available in:

  HTML

  Instructions: Please click on the link above, and work through each of the three examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

  Completing this assessment should take approximately 30 minutes.

  Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

2.3 Volume of Solids of Revolution *When we are presented with a solid that was produced by rotating a curve around an axis, there are two sensible ways to take that solid apart: slice it thinly perpendicularly to the axis, into disks (or washers, if the solid had a hole in the middle), or peel layers from around the outside like the paper wrapper of a crayon. The latter method is known as the shell method and produces thin cylinders. In both cases, we find the area of the thin segments and add them up to find the volume; as usual, when we have infinitely many pieces, this "addition" is really integration.*

2.3.1 Disks and Washers - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 6: Applications of the Integral: "Section 6.2: Volumes of Solids of Revolution" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 6: Applications of the Integral: "Section 6.2: Volumes of Solids of Revolution" (PDF)

```
 Instructions: Please click on the link above, and read Section 6.2
in its entirety (pages 308 through 318).  This reading will cover
sub-subunits 2.3.1-2.3.2.

 Studying this reading should take approximately 1 hour.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
```

```
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Lecture: YouTube: MIT: David Jerison's "Lecture 22: Volumes by Disks and Shells"** Link: YouTube: MIT: David Jerison's "Lecture 22: Volumes by Disks and Shells" (YouTube)

  Also Available in:
  iTunes U

  Instructions: Please click on the link above, and watch the entirety of this video. Note that lecture notes are available in PDF; the link is on the same page as the lecture. Dr. Jerison elaborates on some tangential material for a few minutes in the middle, but returns to the essential material very quickly. This lecture will cover the topics outlined for sub-subunits 2.3.1 and 2.3.2.

  Viewing this lecture and pausing to take notes should take approximately 1 hour.

  Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Aaron Robertson and Dan Birmajer's "Solid of Revolution – Washers"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Aaron Robertson and Dan Birmajer's "Solid of Revolution – Washers" (HTML)

  Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Applications of Integration," and click button 119 (Solid of Revolution – Washers). Do problems 1-12. If at any time a problem set seems too easy for you, feel free to move on.

  Completing this assessment should take approximately 1 hour.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

2.3.2 Cylindrical Shells - Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Aaron Robertson and Dan Birmajer's "Solid of Revolution – Shells" Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Aaron Robertson and Dan Birmajer's "Solid of Revolution – Shells" (HTML)

```
 Instructions: Click on the link above.  Then, click on the "Index"
button.  Scroll down to "2. Applications of Integration," and click
button 120 (Solid of Revolution – Shells).  Do problems 5-17.  If at
any time a problem set seems too easy for you, feel free to move
on.

 Completing this assessment should take approximately 1 hour.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

- **Assessment: Math Centre's "Volumes: Exercises"** Link: Math Centre's "Volumes: Exercises" (Flash)

Instructions: This assessment is for subunits 2.2 and 2.3; do not complete this assessment until you have worked through these subunits in their entirety. Click on the link above, and work through the exercises using the method you feel is most appropriate.

Completing this assessment should take approximately 30 minutes.

Terms of Use: The resource above is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 2.0 UK: England & Wales License (HTML). It is attributed to Math Centre, and the original version can be found here (Flash).

2.4 Lengths of Curves *In this subunit, we will make use of another concept that you have known and understood for quite some time: the distance formula. If you want to estimate the length of a curve on a certain interval, you can simply calculate the distance between the initial point and terminal point using the traditional formula. If you want to increase the accuracy of this measurement, you can identify a third point in the middle and calculate the sum of the two resulting distances. As we add more points to the formula, our accuracy increases: the exact length of the curve will be the sum (i.e. the integral) of the infinitesimally small distances.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 6: Applications of the Integral: "Section 6.3: Length of a Curve"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 6: Applications of the Integral: "Section 6.3: Length of a Curve" (PDF)

Instructions: Please click on the link above, and read Section 6.3 in its entirety (pages 319 through 325). This reading discusses how to calculate the length of a curve, also known as arc length. This includes calculating arc length for parametrically-defined curves.

Studying this reading should take approximately 45 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: David Jerison's "Lecture 31: Parametric Equations, Arclength, Surface Area"** Link: YouTube: MIT: David Jerison's "Lecture 31: Parametric Equations, Arclength, Surface Area" (YouTube)

Also Available in:
iTunes U

Instructions: Please watch this video lecture from the beginning up to time 26:10 minutes. Note that lecture notes are available in PDF; the link is on the same page as the lecture.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Daniel Russo's "Arc Length"** Link: Temple University: Gerardo

Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Daniel Russo's "Arc Length" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Applications of Integration," and click button 125 (Arc Length). Do all problems (1-9). If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

2.5 Surface Areas of Solids *In this subunit, we will combine what we learned earlier in this unit. Though you might expect that calculating the surface area of a solid will be as easy as finding its volume, it actually requires a number of additional steps. You will need to find the curve-length for each of the "slices" we identified earlier and then add them together.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 6: Applications of the Integral: "Section 6.4: Area of a Surface of Revolution"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 6: Applications of the Integral: "Section 6.4: Area of a Surface of Revolution" (PDF)

Instructions: Please click on the link above, and read Section 6.4 in its entirety (pages 327 through 335). In this beautiful presentation of areas of surfaces of revolution, the author again makes use of rigorously-defined infinitesimals, as opposed to limits. Recall that the approaches are equivalent; using an infinitesimal is the same as using a variable and then taking the limit as that variable tends to zero.

Studying this reading should take approximately 1 hour.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: David Jerison's "Lecture 31: Parametric Equations, Arclength, Surface Area"** Link: YouTube: MIT: David Jerison's "Lecture 31: Parametric Equations, Arclength, Surface Area" (YouTube)

Also Available in:
iTunes U

Instructions: Please watch this video lecture from time 26:10 minutes to time 40:35. Note that lecture notes are available in PDF; the link is on the same page as the lecture.

Viewing this lecture and pausing to take notes should take approximately 15-20 minutes.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Areas of Surfaces of Revolution"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Areas of Surfaces of Revolution" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the three examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 15-20 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

2.6 Average Value of Functions *Note: You probably learned about averages (or mean values) quite some time ago. When you have a finite number of numerical values, you add them together and divide by the number of values you have added. There is nothing preventing us from seeking the average of an infinite number of values (i.e. a function over a given interval). In fact, the formula is intuitive: we add the numbers using an integral and divide by the range.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 6: Applications of the Integral: "Section 6.5: Averages"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 6: Applications of the Integral: "Section 6.5: Averages" (PDF)

Instructions: Please click on the link above, and read Section 6.5 in its entirety (pages 336 through 340).

Studying this reading should take approximately 15-20 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: David Jerison's "Lecture 23: Work, Average Value, Probability"** Link: YouTube: MIT: David Jerison's "Lecture 21: Applications of Logarithms and Geometry" (YouTube)

Also Available in:
iTunes U

Instructions: Please watch this video lecture from the beginning up to time 30:00 minutes. Note that lecture notes are available in PDF; the link is on the same page as the lecture. In this lecture, Professor Jerison will explain how to calculate average values and weighted average values.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Daniel Russo's "Average Value of a Function"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Daniel Russo's "Average Value of a Function" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "4. Assorted Application," and click button 124 (Average Value). Do problems 3-11. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

2.7 Physical Applications *We will now apply what we have learned about integration to various aspects of science. You may know that in physics, we calculate "work" by multiplying the force of the work by the distance over which it is exerted. You may also know that density is related to mass and volume. But we now know that distance and volume are very much related to integration. In this subunit, we will explore these and other connections.*

2.7.1 Distance - Reading: Whitman College: David Guichard's Calculus: Chapter 9: Applications of Integration: "Section 9.2: Distance, Velocity, Acceleration" Link: Whitman College: David Guichard's *Calculus*: Chapter 9: Applications of Integration: "Section 9.2: Distance, Velocity, Acceleration" (PDF)

```
 Instructions: Please click on the link above, and read the Section
9.2 in its entirety (pages 192 through 194).

 Studying this reading should take approximately 15-20 minutes.

 Terms of Use: The linked material above has been reposted by the
kind permission of David Guichard, and can be viewed in its original
form
[here](http://www.whitman.edu/mathematics/calculus/calculus_09_Applicat
ions_of_Integration.pdf#page=6)
(PDF).  Please note that this material is under copyright and cannot
be reproduced in any capacity without explicit permission from the
copyright holder.
```

- **Web Media: UC College Prep's *Calculus BC II for AP*: "Applications of Integrals: Displacement Versus Total Distance"** Link: UC College Prep's *Calculus BC II for AP*: "Application of Antiderivatives & Definite Integrals" (YouTube)

Instructions: Click on the link above, and watch the interactive lecture. You may want to have a pencil and paper close by, as you will be prompted to work on related problems during the lecture.

Viewing this lecture should take approximately 45 minutes.

Terms of Use: The resource above is released under a Creative Commons Attribution-NonCommercial-NoDerivs License. It is attributed to the University of California College Prep.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Displacement vs. Distance Traveled"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Displacement vs. Distance Traveled" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the three examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 30 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

2.7.2 Mass and Density - Reading: University of Wisconsin: H. Jerome Kiesler's Elementary Calculus 6.6 "Some Applications to Physics" Link: University of Wisconsin: H. Jerome Kiesler's *Elementary Calculus* 6.6 "Some Applications to Physics" (PDF)

Instructions: Please click on the above link and read the indicated section (pages 341-351).

```
 Studying this reading should take approximately 1 hour.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Web Media: UC College Prep's Calculus BC II for AP: "Applications of Integrals: Center of Mass and Density"** Link: UC College Prep's *Calculus BC II for AP*: "Applications of Integrals: Center of Mass and Density" (YouTube)

Also Available in:

Java

Instructions: Click on the link above and watch the interactive lecture. You may want to have a pencil and paper close by, as you will be prompted to work on related problems during the lecture.

Viewing this lecture should take approximately 45 minutes.

Terms of Use: The resource above is released under a Creative Commons Attribution-NonCommercial-NoDerivs License 3.0 (HTML). It is attributed to The Regents of the University of California and the original version can be found here (Java).

2.7.3 Moments - Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Moments and Centers of Mass" Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Moments and Centers of Mass" (PDF)

```
 Also Available in:

[HTML](http://faculty.eicc.edu/bwood/math150supnotes/supplemental30.htm
l)
```

```
 Instructions: This assessment will test you on what you learned in
sub-subunits 2.7.2 and 2.7.3.  Please click on the link above, and
work through each of the four examples on the page.  As in any
assessment, solve the problem on your own first.  Solutions are
given beneath each example.

 Completing this assessment should take approximately 30 minutes.

 Terms of Use: The material above has been reposted by the kind
permission of Elizabeth Wood, and can be viewed in its original
form [here](http://faculty.eicc.edu/bwood/math150supnotes/supplemental3
0.html) (HTML).
Please note that this material is under copyright and cannot be
reproduced in any capacity without explicit permission from the
copyright holder.
```

2.7.4 Work - Reading: Whitman College: David Guichard's Calculus: Chapter 9: Applications of Integration: "Section 9.5: Work" Link: Whitman College: David Guichard's *Calculus*: Chapter 9: Applications of Integration: "Section 9.5: Work" (PDF)

```
 Instructions: Please click on the link above, and read Section 9.5
in its entirety (pages 205 through 208).  Work is a fundamental
concept from physics roughly corresponding to the distance travelled
by an object multiplied by the force required to move it that
distance.

 Studying this reading should take approximately 30 minutes.

 Terms of Use: The linked material above has been reposted by the
kind permission of David Guichard, and can be viewed in its original
form
[here](http://www.whitman.edu/mathematics/calculus/calculus_09_Applicat
ions_of_Integration.pdf#page=20) (PDF).
Please note that this material is under copyright and cannot be
reproduced in any capacity without explicit permission from the
copyright holder.
```

- **Web Media: UC College Prep's Calculus BC II for AP: "Applications of Integrals: Work Done Moving an Object"** Link: UC College Prep's *Calculus BC II for AP*: "Applications of Integrals: Work Done Moving an Object" (Youtube)

Also Available in:

Java

Instructions: Click on the link above, and watch the interactive lecture. You may want to have a pencil and paper close by, as you will be prompted to work on related problems during the lecture.

Completing this resource should take approximately 30 minutes.

Terms of Use: This video is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives License (HTML). It is attributed to University of California College Prep, and the original version can be found here (Java).

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Work, Fluid Pressures, and Forces"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Work, Fluid Pressures, and Forces" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the seven examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 1 hour.

Unit 3: Techniques and Principles of Integration *Until now, we have been spending the majority of our time on the integration of relatively simple functions (at least in comparison to some of the functions we discussed in Part I). In this unit, we will learn how to analyze more complex functions using more sophisticated machinery. This includes clever methods of substitution, guides to algebraic simplification, and integration by parts, as well as using tables of integration or approximating the integral numerically. We will also address how to manage integrals where either the integrand is discontinuous in the domain of integration, or the domain of integration is infinite.*

Unit 3 Time Advisory
This unit should take you 26.75 hours to complete.

☐ Subunit 3.1: 17.25 hours

☐ Sub-subunit 3.1.1: 3.25 hours

☐ Sub-subunit 3.1.2: 4 hours

☐ Sub-subunit 3.1.3: 2 hours

☐ Sub-subunit 3.1.4: 3.25 hours

☐ Sub-subunit 3.1.5: 4.75 hours

☐ Subunit 3.2: 1 hour

☐ Subunit 3.3: 2.75 hours

☐ Subunit 3.4: 5.75 hours

☐ Sub-subunit 3.4.1: 4.25 hours

☐ Sub-subunit 3.4.2: 1.5 hours

Unit3 Learning Outcomes
Upon successful completion of this unit, the student will be able to: - Use integration by parts to compute definite and indefinite integrals. - Integrate trigonometric functions. - Use trigonometric substitution to compute definite and indefinite integrals. - Use the natural logarithm in substitutions to compute integrals. - Integrate rational functions using the method of partial fractions. - Compute integrals using integral tables. - Approximate integrals using numerical integration techniques including the trapezoidal rule and Simpson's rule. - Compute improper integrals of both types.

3.1 Methods of Integration *So far, we have seen two ideas for computing integrals: directly apply a formula (adjusting for coefficients and using the sum rule to break apart polynomials and similar); or rewrite the integral in some way, either by algebraic manipulation or by substitution. This subunit expands the number of rewriting techniques at our disposal and adds a new technique entirely: integration by parts.*

Unlike differentiation, integration requires a little more forethought or "creativity" in certain situations, as the correct implementation of integration methods is less obvious. In fact, there are often multiple correct methods to solve a complicated integral problem, though they may vary in difficulty. It may not be clear whether the approach you are using is correct until you are partway through the problem, so stick out your attempt until you either succeed or hit an obvious wall.

3.1.1 Integration by Parts *Though this formula may at first seem arbitrary, integration by parts is merely the product rule in reverse. With it, we use the information we have to determine what the initial functions were. Integration by parts is useful for integrands that are the product of two functions from different "families," such as an exponential with a polynomial.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.4: Integrals by Parts"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.4: Integration by Parts" (PDF)

  Instructions: Please click on the link above, and read Section 7.4 in its entirety (pages 391 through 395). Integration by parts is a technique used to integrate more complicated combinations of functions. It is easy to derive – simply rearrange the product rule! Careful bookkeeping is essential for mastering this technique, so keep plenty of scrap paper on hand, use different variables if you have to perform integration by parts a second time in the same problem, and be neat. It will save you time in the end.

  Studying this reading should take approximately 30 minutes.

  Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: David Jerison's "Lecture 30: Integration by Parts, Reduction Formulae"** Link: YouTube: MIT: David Jerison's "Lecture 30: Integration by Parts, Reduction Formulae" (YouTube)

  Also Available in:
  iTunes U

Instructions: Please watch the segment of Lecture 30 from time 18:20 minutes through the end. Note that lecture notes are available in PDF; the links are on the same pages as the lectures.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4)

- **Assessment: University of California, Davis: Duane Kouba's "The Method of Integration by Parts: Problems 1-23"** Link: University of California, Davis: Duane Kouba's "The Method of Integration by Parts: Problems 1-23" (HTML)

Instructions: Click on the link above and work through all of the assigned problems. When you are done, check your solutions with the answers provided.

Completing this assessment should take approximately 2 hours.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.1.2 Trigonometric Integration *Trigonometric identities will be used heavily in this and the next sub-subunit. If you wish to review trigonometric identities, they are covered in the first three sections of Unit 3 of Precalculus II (MA003). Pay particular attention to* $\sin^2 x + \cos^2 x = 1$ *and its counterparts for tan/sec and cot/csc, the half-angle formulas, and the double-angle formulas.*

Trigonometric integration is a simplification method: if you are asked to integrate $\sin^8 x \cos x$, *you can substitute for sin x and be on your way. However, the situation is different if cos x also has a larger exponent. This sub-subunit covers methods to "whittle down" the exponents of such a problem until substitution applies.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.5: Integrals of Powers of Trigonometric Functions"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.5: Integrals of Powers of Trigonometric Functions" (PDF)

Instructions: Please click on the link above, and read Section 7.5 in its entirety (pages 397 through 401).

Studying this reading should take approximately 30 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: Haynes Miller's "Lecture 27: Trigonometric Integrals and Substitution" and "Lecture 28: Integration by Inverse Substitution; Completing the Square"** Link: YouTube: MIT: Haynes Miller's "Lecture 27: Trigonometric Integrals and Substitution" (YouTube)

Also Available in:
iTunes U

and "Lecture 28: Integration by Inverse Substitution; Completing the Square" (YouTube)

Also Available in:
iTunes U

Instructions: Please click on the first link above, and watch the entirety of Lecture 27. Please watch Lecture 28 from the beginning up to time 15:50 minutes. Note that lecture notes are available in PDF; the links are on the same pages as the lectures. In these lectures, Dr. Miller will discuss how to integrate powers of trigonometric functions. At the end of Lecture 27, he will do an example related to trigonometric substitution, which will be the focus of the next section.

Viewing these lectures and pausing to take notes should take approximately 1 hour and 15 minutes.

Terms of Use: The videos above are released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). They are attributed to MIT and the original versions can be found here (Flash or MP4) and here (Flash or MP4).

- **Assessment: University of California, Davis: Duane Kouba's "The Integration of Trigonometric Integrals: Problems 1-27"** Link: University of California, Davis: Duane Kouba's "The Integration of Trigonometric Integrals: Problems 1-27" (HTML)

Instructions: Click on the link above, and work through all of the assigned problems. When you are done, check your solutions with the answers provided.

Completing this assessment should take approximately 2 hours and 15 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.1.3 Trigonometric Substitution *Trigonometric substitution is a particular "inverse substitution" technique. In substitution as we have most commonly seen so far, the new variable is given as a function of the old variable; in inverse substitution, this relationship is reversed. In trigonometric substitution, we let our old variable be a trigonometric function of the new variable, chosen so the Pythagorean Theorem applies to simplify the original integrand. This technique is useful when you have a binomial you wish were a monomial, typically because your binomial is on the bottom of a fraction or under a radical.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.6: Trigonometric Substitutions"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.6: Trigonometric Substitutions" (PDF)

Instructions: Please click on the link above, and read Section 7.6 in its entirety (pages 402 through 405).

Studying this reading should take approximately 15-20 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: Haynes Miller's "Lecture 28: Integration by Inverse Substitution; Completing the Square"** Link: YouTube: MIT: Haynes Miller's "Lecture 28: Integration by Inverse Substitution; Completing the Square" (YouTube)

Also Available in:
iTunes U

Instructions: Please watch Lecture 28 from time 15:50 minutes to the end. Note that lecture notes are available in PDF; the link is on the same page as the lecture.

Viewing this video and pausing to take notes should take approximately 45 minutes.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Dan Reich's "Methods of Integration"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Dan Reich's "Methods of Integration" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "4. Methods of Integration," and click button 164 (Trigonometric Substitution). Do problems 1-11. Note that the first problem has been done for you as an example; just click through the example in order to get a sense of the setup of the module. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.1.4 More Techniques Using the Natural Logarithm *If $y = f(x)$, then $\ln y = \ln (f(x))$. If $f(x)$ involves powers, products, or quotients, taking the natural logarithm may allow us to simplify, making differentiation easier. On the other hand, the fact that the integral of du/u is $\ln u$ seems only narrowly applicable. However, certain functions that do not originally appear to be of the form du/u may be manipulated into that form. In particular, this approach allows us to integrate powers of tangent and secant more easily.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 8: Exponential and Logarithmic Functions: "Section 8.7: Derivatives and Integrals Involving ln(x)"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.7: Derivatives and Integrals Involving ln(x)" (PDF)

Instructions: Please click on the link above, and read Section 8.7 in its entirety (pages 469 through 473).

Studying this reading should take approximately 30 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: The Natural Logarithmic Function"** Link: University of Houston: Selwyn Hollis's "Video Calculus: The Natural Logarithmic Function" (QuickTime)

Instructions: Please click on the link, scroll down to Video 31: "The Natural Logarithmic Function," and watch from the 6th slide (marked 6 of 8) to the end. Feel free to watch the

entire video if you would like a refresher on some earlier concepts. This short video gives examples of how to use the properties of the natural log to compute some more complicated integrals.

Studying this lecture should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: University of California, Davis: Duane Kouba's "The Integration of Rational Functions, Resulting in Logarithmic of Arctangent Functions: Problems 1-22"** Link: University of California, Davis: Duane Kouba's "The Integration of Rational Functions, Resulting in Logarithmic or Arctangent Functions: Problems 1-22" (HTML)

Instructions: This assessment will cover sub-subunits 3.1.3 and 3.1.4. Click on the link above, and work through all of the assigned problems. When you are done, check your solutions with the answers provided.

Completing this assessment should take approximately 2 hours and 15 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.1.5 Rational Functions and Integration with Partial Fractions *You first learned about partial fractions in precalculus, when you learned to re-write the quotient of two rational functions when the denominator function can be written as the product of smaller factors. Here, we use these methods to split apart a complicated fraction into the sum of simpler fractions – in particular, fractions we know how to integrate.*

Polynomial factoring and long division will be used in this sub-subunit. For a review of those topics, see Beginning Algebra (MA001), Unit 4 and subunit 3.4, respectively.

- **Reading: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan's and Donald J. Lewis's Calculus and Linear Algebra Vol. 1: "4-10 Partial Fractions Expansions of Rational Functions"** Link: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan's and Donald J. Lewis's *Calculus and Linear Algebra Vol.1*: "4-10 Partial Fractions Expansions of Rational Functions" (HTML)

Instructions: Please click on the link above, and read the assigned section. Use the "previous" and "next" links at the bottom of the webpage to navigate through this reading. This reading explains how to expand a rational function in terms of fractions of simpler rational functions. If you are not familiar with this method, this reading will be essential for understanding the rest of this section.

Studying this reading should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 8: Exponential and Logarithmic Functions: "Section 8.8: Integration of Rational Functions"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.8: Integration of Rational Functions" (PDF)

Instructions: Please click on the link above, and read Section 8.8 in its entirety (pages 474 through 480). Once you have mastered partial fractions, there are established methods for integrating rational functions. Careful, neat work will help you enormously in this section.

Studying this reading should take approximately 45 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: David Jerison's "Lecture 29: Partial Fractions" and "Lecture 30: Integration by Parts, Reduction Formulae"** Link: YouTube: MIT: David Jerison's "Lecture 29: Partial Fractions" (YouTube)

Also Available in:
iTunes U

and "Lecture 30: Integration by Parts, Reduction Formulae" (YouTube)

Also Available in:
iTunes U

Instructions: Please click on the first link above, and watch the entirety of Lecture 29. Please watch the segment of Lecture 30 from the beginning to time 18:20. Note that lecture notes are available in PDF; the links are on the same pages as the lectures.

Viewing these lectures and pausing to take notes should take approximately 1 hour and 30 minutes.

Terms of Use: The videos above are released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). They are attributed to MIT and the original versions can be found here (Flash or MP4) and here (Flash or MP4).

- **Assessment: University of California, Davis: Duane Kouba's "The Method of Integration by Partial Fractions: Problems 1-20"** Link: University of California, Davis: Duane Kouba's "The Method of Integration by Partial Fractions: Problems 1-20" (HTML)

Instructions: Click on the link above, and work through all of the assigned problems. You will need to scroll down the page a bit to get to the problems. When you are done, check your solutions with the answers provided.

Completing this assessment should take approximately 2 hours.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.2 Integration with Tables & CAS (Computer Algebra Systems) *In theory, one could use the techniques we have learned so far to integrate any integrable function. However, using tables allows us to avoid "reinventing the wheel" and re-deriving the techniques for more complicated but standard integrands by summarizing the results of the process.*

- **Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Calculus II: "Integration Techniques: Using Integral Tables"** Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Calculus II*: "Integration Techniques: Using Integral Tables" (HTML)

Instructions: Please click the link above and read this entire section. In this section, Professor Dawkins gives some helpful hints about using integral tables to compute integrals quickly; this involves reducing whatever problem you are faced with to a problem in the tables. He bases

his discussion on the tables in the textbook used by his classes. However, it is easy to find tables of integrals on the Internet; we list several at the end of this page.

Studying this reading should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.3 Numerical Integration *With numerical integration, we abandon the antiderivative and work with modifications of the Riemann sum. This is possible to do for any integrand, whereas antiderivatives do not exist for every integrand. In most cases, we may also compute an error bound, allowing us to approximate the definite integral to any degree of accuracy we like (provided we have time to carry out all of the computations).*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 4: Integration: "Section 4.6 "Numerical Integration"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 4: Integration: "Section 4.6: Numerical Integration" (PDF)

Instructions: Please click on the link above, and read Section 4.6 in its entirety (pages 224 through 233). In MA101, we used this text to introduce the trapezoidal rule. You will review that method for numerical integration and also learn about Simpson's Rule in this reading. These approximation methods are used by mathematical software to calculate the values of definite integrals to very high degrees of accuracy.

Studying this reading should take approximately 1 hour.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: YouTube: MIT: David Jerison's "Lecture 24: Numerical Integration" and "Lecture 25: Exam Review"** Link: YouTube: MIT: David Jerison's "Lecture 24: Numerical Integration" (YouTube)

Also Available in:
iTunes U

and "Lecture 25: Exam Review" (YouTube)

Also Available in:
iTunes U

Instructions: Please click on the first link above, and watch the segment of Lecture 24 from time 33:50 minutes through the end. Then, click on the second link above, and watch Lecture 25 from the beginning up to time 14:04 minutes. Note that lecture notes are available in PDF; the links are on the same pages as the lectures. In these lectures, Dr. Jerison will discuss motivation and methods for numerical integration, including the trapezoidal rule and Simpson's rule.

Viewing these lectures and pausing to take notes should take approximately 45 minutes.

Terms of Use: The videos above are released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). They are attributed to MIT and the original versions can be found here (Flash or MP4) and here (Flash or MP4).

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Numerical Integration"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Numerical Integration" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the six examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 1 hour.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

3.4 Improper Integrals *In previous integration examples, we have either ignored the domain of the function (i.e. antiderivatives) or integrated over an interval without any discontinuities. But what if we want to integrate over an interval tending toward infinity, or what if we want to find the area under a curve on an interval with a vertical asymptote? This subunit will introduce you to these integrals, which we refer to as "improper integrals."*

3.4.1 Type I - Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Calculus II: "Integration Techniques: Improper Integrals" Link: Lamar University: Paul Dawkin's *Paul's Online Math Notes: Calculus II*: "Integration Techniques: Improper Integrals" (HTML)

```
 Instructions: This reading will cover sub-subunits 3.4.1 and
3.4.2.  Please click the link above, and read this entire section.
 The two types of improper integrals are Type I, those with
"infinite" limits of integration, and Type II, those where the
integrand has infinite discontinuities somewhere in the interval of
integration.  Professor Dawkins does not use this terminology,
although it is common, but he does discuss how to deal with each
type of improper integral.

 Studying this reading should take approximately 1 hour.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

- **Lecture: YouTube: MIT: David Jerison's "Lecture 36: Improper Integrals" and "Lecture 37: Infinite Series and Convergence Tests"** YouTube: MIT: David Jerison's "Lecture 36: Improper Integrals" (YouTube)

Also Available in:
iTunes U

and "Lecture 37: Infinite Series and Convergence Tests" (YouTube)

Also Available in:
iTunes U

Instructions: These lectures will cover sub-subunits 3.4.1 and 3.4.2. Please click on the first link above, and watch Lecture 36 from time 3:22 minutes to the end. Please click on the second link above, and watch Lecture 37 from the beginning up to time 17:35 minutes. Note that lecture notes are available in PDF; the link is on the same page as the lecture. In these lectures, Professor Jerison will discuss how to estimate as well as evaluate improper integrals of both types.

Viewing these lectures and pausing to take notes should take approximately 1 hour and 15 minutes.

Terms of Use: The videos above are released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). They are attributed to MIT and the original versions can be found here (Flash or MP4) and here (Flash or MP4).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Matthias Beck and Molly M. Cow's "Improper Integrals over Unbounded Intervals"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Matthias Beck and Molly M. Cow's "Improper Integrals over Unbounded Intervals" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "6. Improper Integrals," and click button 172 (Improper Integrals over Unbounded Intervals). Do problems 1-18. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 2 hours.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

3.4.2 Type II - Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Matthias Beck and Molly M. Cow's "Improper Integrals of Unbounded Functions" Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Matthias Beck and Molly M. Cow's "Improper Integrals of Unbounded Functions" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "6. Improper Integrals," and click button 173 (Improper Integrals of Unbounded Functions). Do all 16 problems. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour and 30 minutes.

Unit 4: Parametric Equations and Polar Coordinates *So far we have worked in Cartesian (rectangular) coordinates where there has been one dependent variable, say x, and one dependent variable y=f(x). At times this has been inconvenient. Think about the equation describing the graph of a circle,* $x^2 + y^2 = r^2$*: here, y cannot be given as an explicit function of x. For situations like this one there are other ways of describing graphs which make calculations much simpler.*

You studied parametric equations and polar coordinates in subunits 4.2 and 4.5 of Precalculus II (MA003)*, so for an alternate approach, you may review those subunits. Make sure to come back to this unit, because MA003 does not cover as much material.*

Unit 4 Time Advisory
This unit should take you 12.5 hours to complete.

- ☐ Subunit 4.1: 3.5 hours
- ☐ Subunit 4.2: 4 hours
- ☐ Reading: 0.75 hours
- ☐ Lecture: 0.75 hours
- ☐ Assessment: 2.5 hours
- ☐ Subunit 4.3: 2.5 hours
- ☐ Subunit 4.4: 1.25 hours
- ☐ Subunit 4.5: 1.25 hours

Unit4 Learning Outcomes
Upon successful completion of this unit, the student will be able to: - Graph parametric equations. - Find derivatives of parametric equations. - Convert between Cartesian and polar coordinates. - Graph equations in polar coordinates. - Compute derivatives of equations in polar coordinates. - Compute areas under curves described by polar coordinates. - Compute arc length for curves given in polar coordinates.

4.1 Parametric Equations and Their Derivatives *Parametric equations treat x and y each as functions of a third variable, typically t. It is helpful to think of t as time, and the equations as instructing how a curve is to be drawn, giving the pen's coordinates for each point in time. This easily extends to curves in three-dimensional space by adding an equation for z as a function of t.*

- **Reading: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan's and Donald J. Lewis's Calculus and Linear Algebra Vol. 1: "3-9 Parametric Equations"** Link: University of Michigan's Scholarly Monograph Series: Wilfred Kaplan's and Donald J. Lewis's *Calculus and Linear Algebra Vol.1*: "3-9 Parametric Equations" (HTML)

  Instructions: Please click on the link above, and read the assigned section. Use the "previous" and "next" links at the bottom of each webpage to navigate through the reading. This reading discusses parametric equations for curves and how to differentiate them. Note the similarity to related rates. Ignore the reference in the first paragraph to vector equations; that is Calculus III material.

  Studying this reading should take approximately 30 minutes.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Lecture: Khan Academy's "Parametric Equations I" and "Parametric Equations II"; MIT: David Jerison's "Lecture 31: Parametric Equations, Arclength, Surface Area" and "Lecture 32: Polar Coordinates; Area in Polar Coordinates"** Khan Academy's "Parametric

Equations I" (YouTube) and "Parametric Equations II" (YouTube); MIT: David Jerison's "Lecture 31: Parametric Equations, Arclength, Surface Area" (YouTube)

Also Available in:
iTunes U

and "Lecture 32: Polar Coordinates; Area in Polar Coordinates" (YouTube)

Also Available in:
iTunes U

Instructions: Please click on the links above to watch Salman Khan's "Parametric Equation I" and "Parametric Equations II." Then, watch Professor Jerison's Lecture 31 from time 40:35 minutes to the end and Lecture 32 from the beginning up to time 22:50 minutes. Note that lecture notes are available in PDF; the link is on the same page as the lecture.

Salman Khan's first video gives a very intuitive example of the concept of parametric curves—two-dimensional motion with a time parameter. The second video shows how to eliminate the parameter and gives a second example. In the lectures from MIT, Professor Jerison will work through a more advanced example and discuss how to calculate arc length for curves expressed by parametric equations.

Viewing these lectures and pausing to take notes should take approximately 1 hour.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Parameterizations of Plane Curves" and "Calculus with Parameterized Curves"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Parameterization of Plane Curves" (PDF) and "Calculus with Parameterized Curves" (PDF)

Also Available in:

HTML ("Parameterization of Plane Curves")

HTML ("Calculus with Parameterized Curves")

Instructions: Please click on the first link above, and work through each of the nine examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example. Do the same with the second link; work through each of the eight examples on the page on your own before checking them with the given solutions.

Completing these assessments should take approximately 2 hours.

4.2 Polar Coordinates *Note: Polar Coordinates are both a different coordinate system to describe two-dimensional space and, when related back to the "x-y plane," a different parameterization for curves in that system. Instead of representing location by horizontal and vertical distance from the origin, we represent it by straight-line distance from the origin and angle from the positive x-axis (measured counter-clockwise).*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.7: Polar Coordinates"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.7: Polar Coordinates" (PDF)

Instructions: Please click on the link above, and read Section 7.7 in its entirety (pages 406 through 411). Polar coordinates use two parameters, angle and radius, to describe the graphs of curves.

Studying this reading should take approximately 45 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0 (HTML). It is attributed to H. Jerome Kiesler and the original version can be found here (PDF).

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Polar Coordinates and Graphs"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Polar Coordinates and Graphs" (QuickTime)

Instructions: This lecture will cover subunits 4.2 and 4.3. Please click on the link, scroll down to Video 41: "Polar Coordinates and Graphs," and watch the entire lecture. In this video, you will learn how to graph a number of well-known figures in polar coordinates, such as cardioids, roses, and limaçons. You will also learn more about derivatives and tangent lines in polar coordinates.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Dan Reich's "Plotting Points in Polar Coordinates"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Dan Reich's "Plotting Points in Polar Coordinates" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "5. Geometry, Curves, and Polar Coordinates," and click button 181 (Sketching Polar Curves). Do five of the problems in the module as they are presented to you. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Polar Coordinates"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Polar Coordinates" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the eighteen examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 2 hours.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

4.3 Derivatives and Curve Sketching in Polar Coordinates - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.8: Slopes and Curve Sketching in Polar Coordinates" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.8: Slopes and Curve Sketching Polar Coordinates" (PDF)

```
 Instructions: Please click on the link above, and read Section 7.8
in its entirety (pages 412 through 419).  Here, you will learn tips
and tricks for graphing equations in polar coordinates and discover
how to take derivatives of such functions.

 Studying this reading should take approximately 45 minutes.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Activity: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Dan Reich's "Sketching Curves in Polar Coordinates"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Dan Reich's "Sketching Curves in Polar Coordinates" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "5. Geometry, Curves, and Polar Coordinates," and click button 183 (Sketching Polar Curves). Do each of the problems (six in total). This is an exploratory graphing assessment which is more like an applet. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 45 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Graphing in Polar Coordinates"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Graphing in Polar Coordinates" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the eight examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 1 hour.

Terms of Use: The material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

4.4 Areas with Polar Coordinates - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.9: Area in Polar Coordinates" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.9 Area in Polar Coordinates" (PDF)

```
 Instructions: Please click on the link above, and read Section 7.9
in its entirety (pages 420 through 424).

 Studying this reading should take approximately 30 minutes.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Lecture: YouTube: MIT: David Jerison's "Lecture 33: Exam 4 Review"** Link: YouTube: MIT: David Jerison's "Lecture 33: Exam 4 Review" (YouTube)

Also Available in:
iTunes U

Instructions: Please watch Lecture 33 from the beginning to time 34:58. Note that lecture notes in PDF are available for this video; the link is on the same page as the lecture. In this lecture, Professor Jerison will touch on computing area under curves described by polar coordinates and also do several more examples of curve sketching in polar coordinates.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Areas and Length Using Polar Coordinates"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Areas and Lengths Using Polar Coordinates" (QuickTime)

Instructions: This lecture will cover subunits 4.4 and 4.5. Please click on the link, scroll down to Video 42: "Areas and Lengths Using Polar Coordinates," and watch the entire video. This video discusses area (slides 1-5) and arc length (slides 7-9) in polar coordinates.

Viewing this video lecture should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

4.5 Arc Length with Polar Coordinates - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 7: Trigonometric Functions: "Section 7.10: Length of a Curve in Polar Coordinates" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 7: Trigonometric Functions: "Section 7.10: Length of a Curve in Polar Coordinates" (PDF)

```
 Instructions: Please click on the link above, and read Section 7.10
in its entirety (pages 425 through 427).

 Studying this reading should take approximately 15-20 minutes.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Integration in Polar Coordinates"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Integration in Polar Coordinates" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the nine examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example. The examples cover both Arc Length and Area in Polar Coordinates.

Completing this assessment should take approximately 1 hour.

Unit 5: Infinite Sequences and Series *In this unit, you will become acquainted with the infinite lists called sequences and infinite sums called series. The main question for each is whether it converges: do the terms of the sequence have a finite limit? Do the series terms have a finite sum? You will learn ways to test for convergence or divergence. After learning a number of such tests, we will look at Taylor series, which are infinite polynomials. Any function that may be differentiated an unlimited number of times gives rise to a Taylor series, whose partial sums are approximations to the function using ever higher-order derivatives. We will consider questions like: for which values of the variable does the series converge? For those values, is it equal to the function from which it was defined?

Many students find series the most difficult of the topics in Calculus II. There are multiple expositions of each topic included in this unit, so be patient with yourself and study each resource carefully.*

Unit 5 Time Advisory
This unit should take you 29.5 hours to complete.

☐ Subunit 5.1: 4 hours

☐ Reading: 1 hour

- ☐ Lecture: 1 hour
- ☐ Assessment: 2 hours
- ☐ Subunit 5.2: 3.75 hours
- ☐ Subunit 5.3: 8.75 hours
- ☐ Sub-subunit 5.3.1: 1.75 hours
- ☐ Sub-subunit 5.3.2: 2 hours
- ☐ Sub-subunit 5.3.3: 1 hour
- ☐ Sub-subunit 5.3.4: 1.5 hours
- ☐ Sub-subunit 5.3.5: 1.5 hours
- ☐ Sub-subunit 5.3.6: 1 hour
- ☐ Subunit 5.4: 5 hours
- ☐ Sub-subunit 5.4.1: 1.5 hours
- ☐ Sub-subunit 5.4.2-5.4.4: 0.5 hours
- ☐ Sub-subunit 5.4.5: 0.5 hours
- ☐ Sub-subunit 5.4.6: 2.5 hours
- ☐ Subunit 5.5: 11.75 hours
- ☐ Sub-subunit 5.5.1: 4.25 hours
- ☐ Sub-subunit 5.5.2: 0.75 hours
- ☐ Sub-subunit 5.5.3: 4 hours
- ☐ Sub-subunit 5.5.4: 2.75 hours

Unit5 Learning Outcomes
Upon successful completion of this unit, the student will be able to: - Define convergence and limits in the context of sequences. - Define convergence and limits in the context of series. - Find the limits of sequences and series. - Discuss the convergence of the geometric and binomial series. - Show the convergence of positive series using the comparison, integral, limit comparison, ratio, and root tests. - Show the divergence of a positive series using the divergence test. - Show the convergence of alternating series. - Define absolute and conditional convergence. - Show the absolute convergence of a series using the comparison, integral, limit comparison, ratio, and root tests. - Manipulate power series algebraically. - Differentiate and integrate power series. - Compute Taylor and MacLaurin series. - Approximate functions using power series.

5.1 Sequences *A sequence is merely a list of terms (usually numbers) that are arranged in a particular order. In this subunit, we will look at a sequence of numbers ordered by some rule or function.*

- **Reading: Whitman College: David Guichard's Calculus: Chapter 11: Sequences and Series: "Section 11.1 Sequences"** Link: Whitman College: David Guichard's *Calculus*: Chapter 11: Sequences and Series: "Section 11.1: Sequences" (PDF)

Instructions: Please click on the link above, and read the brief introduction to chapter 11 and Section 11.1 (pages 253 through 260).

Studying this reading should take approximately 1 hour.

Terms of Use: The linked material above has been reposted by the kind permission of David Guichard, and can be viewed in its original form here (PDF). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Sequences I" and "Sequences II"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Sequences I" (QuickTime) Lecture and "Sequences II" (QuickTime)

Instructions: Please click on the link, scroll down to Video 47: "Sequences I," and watch the entire video. Next, scroll down to Video 48: "Sequences II," and watch it from the beginning through the 7th slide (the end of the slide marked 7 of 12). If you are interested, feel free to watch the rest of the video.

In the first video, Dr. Hollis discusses sequences and limits, goes over several important limits, and explains growth rates and order comparisons. In the second video, he gives a precise definition of limits, shows how to do an epsilon-N proof for limits of sequences, and discusses *boundedness* and *monotonicity*. In the optional slides (8-12), he discusses recursively-defined sequences, fixed points, and cobweb plots.

Viewing these lectures and note-taking should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "Limits of Sequences"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "Limits of Sequences" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "1. Sequences," and click button 184 (Limits of Sequences). Do problems 1-22. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 2 hours.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.2 Series *A series is the sum of the terms in an infinite sequence. You are likely familiar with series arranged in an arithmetic or geometric progression; this subunit will take a look at terms defined by more intricate functions. You can also view a series as another type of sequence – a sequence of partial sums. In the following readings, you will learn what it means for a series to converge and study some important types of series.*

5.2.1 Series and Basic Convergence - Reading: Whitman College: David Guichard's Calculus: Chapter 11: Sequences and Series: "Section 11.2 Series" Link: Whitman College: David Guichard's *Calculus*: Chapter 11: Sequences and Series: "Section 11.2: Series" (PDF)

```
 Instructions: Please click on the link above, scroll down and read
Section 11.2 in its entirety (pages 260 through 263).  This section
will introduce you to infinite series and make mention of the
geometric series, which will be discussed in more detail below.  It
also explains what it means for a series to converge.

 Studying this reading should take approximately 15-20 minutes.

 Terms of Use: The linked material above has been reposted by the
kind permission of David Guichard, and can be viewed in its original
form
[here](http://www.whitman.edu/mathematics/multivariable/multivariable_1
1_Sequences_and_Series.pdf)
(PDF).  Please note that this material is under copyright and cannot
be reproduced in any capacity without explicit permission from the
copyright holder.
```

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Series"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Series" (QuickTime)

Instructions: This lecture will cover sub-subunits 5.2.1-5.3.1. Please click on the link, scroll down to Video 49: "Series," and watch the entire video. You are welcome to break it into parts as you go along. Slides 1-4 correspond roughly to 5.2.1; slides 5-7 correspond roughly to 5.2.3; slides 8-11 are an exposition of 5.2.2; and slide 12 corresponds to 5.3.1.

Viewing this lecture and note-taking should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.2.2 Properties of Infinite Series - Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Calculus II "Special Series" Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Calculus II*: "Special Series" (HTML)

```
 Instructions: Please click on the link above, and read the
information on this webpage.  You may skip the section on
telescoping series; you are not responsible for that material.
However, pay attention to the beginning through the discussion
following Example 2, pick up again at the paragraph before Example
5, and read to the end.  The notion that any finite number of terms
has no effect on the convergence behavior of a series is important
and can save you a lot of effort.

 Studying this reading should take approximately 30 minutes.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

5.2.3 Focus on the Geometric Series *A series is geometric if every successive term is the product of the previous term with a fixed value called the ratio of the series. For example, 3 + 6*

+ 12 + 24 + ... is a geometric series with ratio 2. Geometric series are unusual in that not only can we easily determine convergence or divergence for them, but in the case of convergence, we can also find the sum of the series exactly.

- **Reading: MIT: Gilbert Strang's Calculus: Chapter 10: Infinite Series: "Section 10.1: The Geometric Series"** Link: MIT: Gilbert Strang's *Calculus*: Chapter 10: Infinite Series: "Section 10.1: The Geometric Series" (PDF)

Instructions: Read the several paragraphs at the beginning of chapter 10 (the discussion of the geometric series begins here) and section 10.1 (pages 366-372).

Studying this reading should take approximately 1 hour.

Terms of Use: The article above is released under Creative Commons Attribution-NonCommercial-ShareAlike 3.0 (HTML). It is attributed to Professor Gilbert Strang (MIT) and the original version can be found here (PDF).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Daniel Russo's "Geometric Series"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Daniel Russo's "Geometric Series" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "1. Integration," and click button 101 (Geometric Series). Do problems 2-10. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.2.4 Highlight: The Binomial Series *If you wish to expand $(1 + x)^2$ or $(1 + x)^3$, you may simply multiply them out. But what are the coefficients of the fifteen terms in the expansion of $(1 + x)^{14}$? The binomial theorem provides an answer in terms of combinations, also known as binomial coefficients. The binomial series takes this even a step further, allowing the expansion of expressions such as $1/(1 + x)$ and $(1 + x)^{1/2}$ to infinite polynomials.*

- **Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Calculus II: "Sequences and Series: Binomial Series"** Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Calculus II*: "Sequences and Series: Binomial Series" (HTML)

Instructions: Please click the link above, and read this entire section. Recall that combinations arise from the problem of counting subsets of a collection of objects: the number of ways to choose two of the four letters ABCD is the combination 4 choose 2, which is 6. If you have not seen it before, the connection to powers of binomials may not be clear. To expand $(1 + x)^2$, you write it as two copies of $1 + x$ and multiply one term from the first copy by one term from the second copy, using every possible pairing exactly once. Likewise, to expand $(1 + x)^4$, you must multiply across the four copies of $(1 + x)$, taking every possible quartet exactly once: all the 1's, the first two 1's and the last two x's, the first two x's and the last two 1's, etc. Combinations allow you to find the coefficients, because, for example, the coefficient of x^2 in the expansion of $(1 + x)^4$ is 6: the number of quartets that contain exactly two x's, or in other words the number of ways to select the two copies of $(1 + x)$ that provide the x's.

The combination k choose n is typically written as the fraction $(k!)/(n!(k-n)!)$. Professor Dawkins writes it the way he does so it will generalize to negative and non-integer values of k.

Studying this reading should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.3 Test for Convergence of Positive Series *You will first learn how to check when a series with only positive terms converges – i.e. the* ***limit*** *of its* ***sequence of partial sums*** *exists and is finite. The theory begins here with* ***positive series****, because it is the simplest problem to consider; these tests often work by "squeezing" the partial sums between zero and some larger series which are known to converge.*

It is important to notice many of the tests related to series are one-directional implications: for example, if the series terms do not limit to zero, you can conclude the series diverges. However, if it lacks that property, then you need more information to conclude either convergence or divergence. Be careful not to assume a lack of one conclusion implies the opposite conclusion.

5.3.1 Divergence Test *Convergence of a series is convergence of its sequence of partial sums. That is, for the series to converge, the partial sums must settle down and overall get closer and closer to a fixed finite value. In order for that to happen, the amount being added to each partial sum to produce the next one must gradually shrink away to nothing. That is the idea of the divergence test, which applies to any series (not just those with all positive terms): if the limit of the terms of the series is not zero, the series diverges.*

This is not an equivalence, however! Many divergent series have terms that limit to zero. The terms must shrink to zero rapidly to give convergence. However, whether the terms shrink to zero at all is straightforward to check and may save you work making more complicated tests on a divergent series.

- **Reading: MIT: Gilbert Strang's Calculus: Chapter 10: Infinite Series: "Section 10.2: Convergence Tests: Positive Series"** Link: MIT: Gilbert Strang's *Calculus*: Chapter 10: Infinite Series: "Section 10.2 Convergence Tests: Positive Series" (PDF)

Instructions: This reading will cover sub-subunits 5.3.1 – 5.3.6. Please click on the link above, and read Section 10.2 in its entirety (pages 374 through 379).

The section presents a criterion for divergence, the integral test, the comparison test, and the ratio and root tests. We will revisit all these tests later in a slightly different context and give a more thorough justification of the last two tests.

Studying this reading should take approximately 45 minutes.

Terms of Use: The article above is released under Creative Commons Attribution-NonCommercial-ShareAlike 3.0 (HTML). It is attributed to Professor Gilbert Strang (MIT) and the original version can be found here (PDF).

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "The Divergence Test"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "The Divergence Test" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Series," and click button 187 (The Divergence Test). Do problems 1-10. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.3.2 Integral Test *There is an imprecise correspondence between sequences and functions as well as between series and integrals. The integral test shows this correspondence; though this relationship is not perfect, it is close enough to be useful.*

- **Lecture: YouTube: MIT: David Jerison's "Lecture 37: Infinite Series and Convergence Tests"** Link: YouTube: MIT: David Jerison's "Lecture 37: Infinite Series and Convergence Tests" (YouTube)

Also Available in:
iTunes U

Instructions: This lecture will cover sub-subunits 5.3.2 - 5.3.4. Please watch Lecture 37 from time 17:35 minutes to the end. Note that lecture notes are available in PDF; the link is on the same page as the lecture. Professor Jerison will begin his discussion of infinite series with the series [Sum of 1 divided by (n squared)]. He introduces important terminology and then proves the convergence of the above series by comparison with the integral of the summand. He extends this argument to state the integral test. Finally, he goes over the limit comparison test for positive sequences.

Viewing this lecture and pausing to take notes should take approximately 45 minutes.

Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: The Integral Test"** Link: University of Houston: Selwyn Hollis's "Video Calculus: The Integral Test" (QuickTime)

Instructions: Please click on the link above, scroll down to Video 50: "The Integral Test," and watch this entire lecture. This short video restates the integral test in a more concise way and provides several other important applications of the integral test, such as proving the convergence of the p-series and estimating remainders of partial sums.

Viewing this lecture should take approximately 15 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "The Integral Test"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "The Integral Test" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Series," and click button 188 (The Integral Test). Do problems 1-10. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.3.3 Comparison Test *Series with no negative terms either have terms that get small enough or fast enough for the series to converge, or terms that remain too large and hence cause the series*

to diverge. It would seem logical that if Series A converges, and the nth term of Series B is less than or equal to the nth term of Series A for all n (at least after a finite number of terms), then Series B should converge: if A's terms are small enough, B's should also be. Likewise, a series with terms that are larger than the corresponding terms of a divergent series should diverge. This is true and is known as the (direct) comparison test.

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Comparison Tests"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Comparison Tests" (QuickTime)

Instructions: This lecture will cover sub-subunits 5.3.3-5.3.6. Please click on the link above, scroll down to Video 51: "Comparison Tests," and watch the video in its entirety. This video states, proves, and applies the comparison, limit comparison, ratio, and root tests for positive series.

Viewing this lecture and pausing to take notes should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Furman University: Dan Sloughter's Difference Equations to Differential Equations: "5.4 Infinite Series: The Comparison Test"** Link: Furman University: Dan Sloughter's *Difference Equations to Differential Equations*: "5.4 Infinite Series: The Comparison Test" (PDF)

Instructions: Please click the link above, and do problems 1 (a, c, e, g), 2 (a, c, e, g), and 4. When finished, click here for solutions (courtesy of the author's blog).

Completing this assessment should take approximately 30 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0. It is attributed to Dan Sloughter and the original version can be found here (HTML). Please respect the copyright and terms of use displayed on the solution guide above.

5.3.4 Ratio Test *The ratio and root tests are two ways of checking whether a series is "geometric in the limit" and thereby drawing conclusions about its behavior. The geometric series with terms ar^n, a and r positive, has the properties that the ratio of the $n+1^{st}$ term to the nth term is always r, and the nth root of the nth term is always r times the nth root of a. The limit of each of those values as n tends to infinity is r. The ratio test and root test check the limits of those values as computed from other series. Although we lose some precision – i.e., a geometric series with ratio 1 diverges, but a limit of 1 in the ratio or root test is inconclusive – these tests greatly increase the number of series for which we can determine convergence or divergence. Typically, only one of these tests is algebraically feasible for a given series, but in the event both are, note that if one is inconclusive, the other will be as well.*

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "The Ratio Test"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "The Ratio Test" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Series," and click button 189 (The Ratio Test). Do problems 4-17. (See the navigation buttons below the problem.) If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour and 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.3.5 Root Test - Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "The nth Root Test" Module Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "The nth Root Test" (HTML)

```
 Instructions: Click on the link above.  Then, click on the "Index"
button.  Scroll down to "2. Series," and click button 190 (The nth
Root Test).  Do problems 3-17.  (See the navigation buttons below
the problem.)  If at any time a problem set seems too easy for you,
feel free to move on.

 Completing this assessment should take approximately 1 hour and 30
minutes.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

5.3.6 Limit Comparison Test *Just as the ratio and root tests check whether a series is "geometric in the limit," the limit comparison test checks whether two series are "equal in the limit," up to a non-zero constant multiple. If so, we can conclude the series have the same convergence behavior.*

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: "The Limit Comparison Test"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "The Limit Comparison Test" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "2. Series," and click button 191 (The Limit Comparison Test). Do problems 1-12. (See the navigation buttons below the problem.) If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's Supplemental Notes for Calculus II: "Ratio and Root Test for Series of Nonnegative Terms" and "Infinite Series"** Link: Clinton Community College: Elizabeth Wood's Supplemental Notes for Calculus II: "Ratio and Root Test for Series of Nonnegative Terms" (PDF) and "Infinite Series" (PDF)

Also Available in:

HTML ("Ratio and Root Test for Series of Nonnegative Terms")

HTML ("Infinite Series")

Instructions: This assessment will cover subunits 5.2-5.3. Please click on the first link above and work through each of the seven examples on the page. Next, please click on the second

link and work through each of the thirteen examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 2 hours.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML) and here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

5.4 Tests for Absolute and Conditional Convergence *Thus far, we have worked with series with all non-negative terms. When series with negative terms are allowed, the picture changes slightly. If all terms are negative, of course, the series is simply -1 times a series of all positive terms and has the same convergence behavior as the positive series. If the terms are mixed sign, however, the positive and negative terms cancel each other out to some degree. For such series, we have essentially two options: the alternating series test, which requires the signs alternate, or to take the absolute value of each term and test that series for convergence.*

5.4.1 Alternating Series Test *An alternating series is one in which the terms alternate between positive and negative. You may view it as a sequence of partial sums that alternately increase and decrease. The alternating series test says that if the magnitudes of the terms decrease to zero in the limit, then the series converges. For example, if every increase or decrease of the partial sums is smaller than the previous, and they limit to zero, the partial sums themselves have a finite limit. Alternating series that are easy to write down tend either to diverge by the divergence test or converge by the alternating series test, but be aware that it is easy to define an alternating series with terms limiting to zero (but not decreasing to zero) that diverges. For example, $1 - 1/2 + 2/3 - 1/3 + 1/2 - 1/4 + \ldots$. This is the harmonic series in disguise, as you will see if you pair off each positive term with the negative term following it.*

- **Reading: Whitman College: David Guichard's Calculus: Chapter 11: Sequences and Series: "Section 11.4: Alternating Series"** Link: Whitman College: David Guichard's *Calculus*: Chapter 11: Sequences and Series: "Section 11.4: Alternating Series" (PDF)

Instructions: Please click on the link above, locate Chapter 11, and read Section 11.4 in its entirety (pages 269 through 273).

Studying this reading should take approximately 30 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of David Guichard, and can be viewed in its original form here (PDF). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

- **Lecture: University of Houston: Selwyn Hollis's "Video Calculus: Alternating Series and Absolute Convergence"** Link: University of Houston: Selwyn Hollis's "Video Calculus: Alternating Series and Absolute Convergence" (QuickTime)

Instructions: This lecture will cover the topics outlined in sub-subunits 5.4.1 and 5.4.2 as well as 5.4.4 and 5.4.5. Please click on the link above, scroll down to Video 52: "Alternating Series and Absolute Convergence," and watch the video lecture in its entirety. This video explains alternating series, conditional and absolute convergence, and the ratio and root tests for absolute convergence.

Viewing this lecture and pausing to take notes should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Alternating Series"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Alternating Series" (PDF)

Also Availabe in:

HTML

Instructions: Please click on the link above, and work through each of the five examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 30 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

5.4.2 Definition of Absolute and Conditional Convergence *As we have seen with the harmonic and alternating harmonic series, the cancellation effect of a mixture of positive and negative terms can be vital for the convergence of a series. The alternating harmonic is called a conditionally convergent series: its convergence is conditional on the cancellation effect. If it is possible to eliminate the cancellation (by taking the absolute value of each term) and still have convergence, the series is called absolutely convergent. When series have a mixture of positive and negative terms but do not alternate sign, taking the absolute value of the terms and testing the resulting series for convergence is often a good method. It cannot tell you if the original series diverges, but it can show if the original series converges, absolutely.*

- **Reading: MIT: Gilbert Strang's Calculus: Chapter 10: Infinite Series: "Section 10.3: Convergence Tests: All Series"** Link: MIT: Gilbert Strang's *Calculus*: Chapter 10: Infinite Series: "Section 10.3: Convergence Tests: All Series" (PDF)

Instructions: This reading will cover sub-subunits 5.4.2 and 5.4.3. Please click on the link above, and read Section 10.3 in its entirety (pages 381 through 384). It is worth pointing out that if a series converges conditionally, it does not have a well-defined sum. By rearranging the terms, the value of the sum can be changed. This is an example of the fact that infinity is a strange place.

Studying this reading should take approximately 30 minutes.

Terms of Use: The article above is released under Creative Commons Attribution-NonCommercial-ShareAlike 3.0 (HTML). It is attributed to Professor Gilbert Strang (MIT) and the original version can be found here (PDF).

5.4.3 Comparison Test for Absolute Convergence *Note: This topic is covered by the reading assigned below sub-subunit 5.4.1.*

5.4.4 Limit Comparison Test for Absolute Convergence *Note: This topic is covered by the lecture assigned below sub-subunit 5.4.1.*

5.4.5 Ratio Test for Absolute Convergence - Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Calculus II: "Sequences and Series: Ratio Test" Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Calculus II*: "Sequences and Series: Ratio Test" (HTML)

```
 Instructions: Please click the link above, and read this entire
section.  This reading defines, applies, and proves the ratio
test*.*

 Studying this reading should take approximately 30 minutes.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

5.4.6 Root Test for Absolute Convergence - Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Calculus II: "Sequences and Series: Root Test" Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Calculus II*: "Sequences and Series: Root Test" (HTML)

```
 Instructions: Please click the link above, and read this entire
section.  This reading defines, applies, and proves the root test.

 Studying this reading should take approximately 30 minutes.

 Terms of Use: Please respect the copyright and terms of use
displayed on the webpage above.
```

- **Assessment: Furman University: Dan Sloughter's Difference Equations to Differential Equations: "5.6 Infinite Series: Absolute Convergence"** Link: Furman University: Dan Sloughter's *Difference Equations to Differential Equations*: "5.6 Infinite Series: Absolute Convergence" (PDF)

Instructions: This assessment will cover sub-subunits 5.4.1-5.4.6. Please click the link above, and do problems 1 (a, c, e, g), 2 (a, c, e, g), and 3 (a, b, c). When finished, click here for solutions (courtesy of the author's blog).

Completing this assessment should take approximately 45 minutes.

Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0. It is attributed to Dan Sloughter and the original version can be found here (HTML). Please respect the copyright and terms of use displayed on the solutions guide above.

- **Assessment: Millersville University: Bruce Ikenaga's "Absolute Convergence and Conditional Convergence"** Link: Millersville University: Bruce Ikenaga's "Absolute Convergence and Conditional Convergence" (HTML)

Instructions: This assessment will cover sub-subunits 5.4.1-5.4.6. Please click on the link above and scroll down the page. Work through the last four examples before looking at their solutions.

Completing this assessment should take approximately 30 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Absolute and Conditional Convergence"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Absolute and Conditional Convergence" (PDF)

Also Available in:

HTML

Instructions: This assessment will cover sub-subunits 5.4.1-5.4.6. Please click on the link above, and work through each of the six examples on the page. As in any assessment, solve the problem on your own first. Detailed solutions are given beneath each example.

Completing this assessment should take approximately 45 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

5.5 Series Representations of Functions *In Single-Variable Calculus I (MA101), we learned that we can approximate a function about a point when we have information about the function's value and its slope at that point. In this subunit, you will learn how to be even more accurate by gathering additional information about the function at a particular point (i.e. by using the second derivative, third derivative, fourth derivative, etc.). The more information you collect, the closer you will get to the function itself. The series representation of a function is the infinite series about a point, taking into consideration all of the derivatives about that point and in the form of a polynomial. This will enable us to look at functions, derivatives, and integrals in new and rather intuitive ways.*

5.5.1 Power Series *A power series is any series where the nth term contains x^n (where x as the name of the variable and the exact matching of the term index with the exponent are not essential). Essentially, it is an infinite polynomial in x.*

- **Reading: Whitman College: David Guichard's Calculus: "Chapter 11: Sequences and Series: Section 11.8: Power Series"** Link: Whitman College: David Guichard's *Calculus*: "Chapter 11: Sequences and Series: Section 11.8: Power Series" (PDF)

Instructions: Please click on the link above and read Section 11.8 in its entirety (pages 278 through 281). A key term to understand in this section is *radius of convergence.*

Studying this reading should take approximately 15-20 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of David Guichard, and can be viewed in its original form here (PDF). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

- **Lecture: YouTube: MIT: David Jerison's "Lecture 38: Taylor Series" and Haynes Miller's "Lecture 39: Final Review"** Link: YouTube: MIT: David Jerison's "Lecture 38: Taylor Series" (YouTube)

Also Available in:
iTunes U

and Haynes Miller's "Lecture 39: Final Review" (YouTube)

Also Available in:
iTunes U

Instructions: These lectures cover subunits 5.5.1-5.5.3. After completing the readings for these sections, click on the first link, and watch Lecture 38 from the 22:45 minute mark to the end. In this lecture, Professor Jerison will discuss general power series before introducing Taylor Series. Then, watch Lecture 39. Professor Miller will continue this exposition; he will go over the derivations for the power series for the exponential, the sine, and the cosine before moving on to other examples.
Note that on the original pages (linked below), the lecture notes are available in PDF under the "Related Resources" tab.

Watching these lectures and pausing to take notes should take approximately 1 hour and 15 minutes.

Terms of Use: The videos above are released under a Creative Commons Attribution-Share-Alike License 3.0. They are attributed to MIT and the original versions can be found here and here.

- **Reading: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Power Series and the Uses of Power Series"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Power Series and the Uses of Power Series" (PDF)

Also Available in:

HTML

Instructions: Please click on the link above, and work through each of the six examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 45 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "Power Series"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "Power Series" (HTML)

Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "3. Power and Taylor Series," and click button 192 (Power Series). Do the problems in the module as they are presented to you (18 total). These problems all deal with computing the radius of convergence for a power series. If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 2 hours.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.5.2 Calculus with Power Series *Happily, our infinite polynomials interact with integration and differentiation in the same way as finite polynomials: the integral of a sum is the sum of the integrals of each term, and likewise for derivatives. The bookkeeping aspects of this topic will be easier if you think of the "point of view" of the mathematical operators: the integral sign and derivative operator d/dx see x as a variable and n as a constant (a different fixed value for each series term). On the other hand, the summation operator sees n as the variable and x as the constant (a value that will be the same for every series term).*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 9: Infinite Series: "Section 9.8: Derivatives and Integrals of Power Series"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 9: Infinite Series: "Section 9.8: Derivatives and Integrals of Power Series" (PDF)

Instructions: Please click on the link above, and read Section 9.8 in its entirety (pages 533 through 539).

Under certain conditions, power series can be differentiated and integrated. Certain characteristics of the series may change, however, such as the interval of convergence. We have not been using Keisler's text so far this unit, because hyperreals are not as helpful to the intuition for series as they are for integrals. In the last several sub-subunits, they do not appear except in the proof of Theorem 1, part (iii), on pages 537-538 of this section. That statement, that the radius of convergence remains the same when a power series is integrated or differentiated, is typically given in calculus texts without proof. Do not fret too much over the proof.

Studying this reading should take approximately 45 minutes to complete.

5.5.3 Taylor and Maclaurin Series *Taylor series are a particular kind of power series, defined from a function. Maclaurin series are a particular kind of Taylor series. The definition allows you to expand any function into an infinite polynomial, which in many cases will be provably equal to the original function, and may be much easier to compute with.*

- **Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 9: Infinite Series: "Section 9.10: Taylor's Theorem" and "Section 9.11: Taylor Series"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 9: Infinite Series: "Section 9.10: Taylor's Theorem" (PDF) and "Section 9.11: Taylor Series" (PDF)

Instructions: Please click on the links above, and read Sections 9.10 and 9.11 (pages 547 through 560).

Taylor Series use the information provided by the derivatives of a function (slope of the tangent line, concavity, etc.) to approximate the function by a sequence of polynomials. Taylor's Theorem tells how good this approximation is. Taylor Series are used extensively in higher mathematics, especially in numerical analysis.

Studying these readings should take approximately 2 hours.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Taylor and MacLaurin Series"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Taylor and MacLaurin Series" (PDF)

  Also Available in:

  HTML

  Instructions: Please click on the link above, and work through each of the seven examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

  Completing this assessment should take approximately 1 hour.

  Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here (HTML). Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book III: Gerardo Mendoza's "Taylor Series"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book III*: Gerardo Mendoza's "Taylor Series" (HTML)

  Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "3. Power and Taylor Series," and click button 193 (Taylor Series). Choose at least 5 of the problems to do. If at any time a problem set seems too easy for you, feel free to move on.

  Completing this assessment should take approximately 1 hour.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

5.5.4 Approximation by Power Series - Reading: University of Wisconsin: H. Jerome Keisler's Elementary Calculus: Chapter 9: Infinite Series: "Section 9.9: Approximations by Power Series" Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 9: Infinite Series: "Section 9.9: Approximations by Power Series" (PDF)

```
 Instructions: Please click on the link above, and read Section 9.9
in its entirety (pages 540 through 546).  Approximation by power
series is a very important topic; for instance, it is how
calculators compute sines and cosines!

 Studying this reading should take approximately 1 hour.

 Terms of Use: The article above is released under a [Creative
Commons Attribution-NonCommercial-ShareAlike License
3.0](http://creativecommons.org/licenses/by-nc-sa/3.0/) (HTML).  It
is attributed to H. Jerome Kiesler and the original version can be
found [here](http://www.math.wisc.edu/~keisler/calc.html) (PDF).
```

- **Web Media: UC College Prep's *Calculus BC II for AP*: "Infinite Sequences and Series: Approximating Functions Using Polynomials" and "Infinite Sequences and Series: Applications of Taylor Series"** Link: UC College Prep's *Calculus BC II for AP*: "Infinite

Sequences and Series: Approximating Functions Using Polynomials" (YouTube) and "Infinite Sequences and Series: Applications of Taylor Series" (YouTube)

Instructions: Click on the links above, and watch the interactive lectures. You may want to have a pencil and paper close by, as you will be prompted to work on related problems during the lecture.

Studying these lectures should take approximately 1 hour.

Terms of Use: The videos above are licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives License. They are attributed to University of California College Prep.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Other Topics Related to Taylor Series"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Other Topics Related to Taylor Series" (PDF)

Also Available in:

HTML

Instructions: Click on the link above, and work through each of the five examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 45 minutes.

Unit 6: Differential Equations *This final unit will introduce the relationship between the mathematical machinery we have been developing and mathematical modeling. In practical situations, you will rarely have all of the information or data needed to represent an initial function. You will likely only have information about how the data changes. In this unit, you will learn how to apply what we know about functions and how they behave in order to model and interpret data.*

Unit 6 Time Advisory
This unit should take you 16.5 hours to complete.

☐ Subunit 6.1: 1.5 hours

☐ Subunit 6.2: 3.5 hours

☐ Subunit 6.3: 4.75 hours

☐ Reading: 2 hours

☐ Lecture: 2 hours

☐ Interactive Lab: 0.5 hours

☐ Assessment: 0.25 hours

☐ Subunit 6.4: 6.75 hours

- ☐ Sub-subunit 6.4.1: 2.75 hours
- ☐ Sub-subunit 6.4.2: 0.5 hours
- ☐ Sub-subunit 6.4.3: 0.5 hours
- ☐ Sub-subunit 6.4.4: 1.5 hours
- ☐ Sub-subunit 6.4.5: 1.5 hours

Unit6 Learning Outcomes
Upon successful completion of this unit, the student will be able to: - Recognize a first order differential equation. - Recognize an initial value problem. - Solve a first order ODE/IVP using separation of variables. - Draw a slope field given an ODE. - Use Euler's method to approximate solutions to basic ODE. - Apply basic solution techniques for linear, first order ODE to problems involving exponential growth and decay, logistic growth, radioactive decay, compound interest, epidemiology, and Newton's Law of Cooling.

6.1 First-Order Differential Equations *A differential equation represents the relationship between an unknown function and its various higher-order derivatives. The order of the relationship is defined by the highest-ordered derivative in the equation. In this subunit, we will only study equations involving an unknown function and its first derivative. We will leave higher-ordered differential equations for later courses.*

- **Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Differential Equations: "Basic Concepts: Definitions"** Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Differential Equations*: "Basic Concepts: Definitions" (HTML)

  Instructions: Please click the link above, and read this entire section. This reading introduces you to the fundamental concepts of differential equations. Key words to remember are: *order, linear differential equation, initial condition,* and *initial value problem.*

  Studying this reading should take approximately 45 minutes.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Differential Equations and Initial Value Problems"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus I: Differential Equations and Initial Value Problems" (PDF)

  Also Available in:

  HTML

  Instructions: Click on the link above and work through examples 1-5 on the page. As in any assessment, solve the problem on your own first. Detailed solutions are given beneath each example.

  Completing this assessment should take approximately 45 minutes.

  Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here. Please note that this material is under

copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

6.2 Separation of Variables and Initial Value Problems *Note: Separation of variables is a method for solving certain types of differential equations. It is based on the assumption that we can "separate" our equation into two pieces: a function of the independent variable and a function of the dependent variable, with no occurrences of one in the other.*

- **Reading: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.6: Some Differential Equations"** Link: University of Wisconsin: H. Jerome Keisler's *Elementary Calculus*: Chapter 8: Exponential and Logarithmic Functions: "Section 8.6: Some Differential Equations" (PDF)

  Instructions: Click on the link above, and read Section 8.6 (pages 461 through 468).The most basic differential equation is the separable equation; in this section, you will learn how to solve such equations.

  Studying this reading should take approximately 1 hour.

  Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0. It is attributed to H. Jerome Kiesler and the original version can be found here.

- **Lecture: YouTube: MIT: David Jerison's "Lecture 16: Differential Equations, Separation of Variables"** Link: YouTube: MIT: David Jerison's "Lecture 16: Differential Equations, Separation of Variables" (YouTube)

  Also Available in:
  iTunes U

  Instructions: Please click on the link above, and watch the segment of this video lecture beginning at 1:50 minutes and ending at 43:20 minutes. Note that lecture notes are available in PDF; the link is on the same page as the lecture. Professor Jerison discusses a number of problems involving separation of variables. His first example is good, but somewhat more complicated than later examples.

  Viewing this lecture and note-taking should take approximately 45 minutes.

  Terms of Use: The video above is released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). It is attributed to MIT and the original version can be found here (Flash or MP4).

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: First Order Differential Equations"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: First Order Differential Equations" (PDF)

  Also Available in:

  HTML

  Instructions: This assessment will cover subunits 6.1-6.2. Click on the link above and work through each of the six examples on the page. As in any assessment, solve the problem on your own first. Solutions are given beneath each example.

Completing this assessment should take approximately 45 minutes.

Terms of Use: The linked material above has been reposted by the kind permission of Elizabeth Wood, and can be viewed in its original form here. Please note that this material is under copyright and cannot be reproduced in any capacity without explicit permission from the copyright holder.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Daniel Hartenstine's "Differential Equations"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Daniel Hartenstine's "Differential Equations" (HTML)

Instructions: This assessment will cover subunits 6.1-6.2. Click on the link above. Then, click on the "Index" button. Scroll down to "2. Applications of Integration," and click button 127 (Differential Equations). Do all problems (1-10). If at any time a problem set seems too easy for you, feel free to move on.

Completing this assessment should take approximately 1 hour.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

6.3 Slope Fields & Euler's Method *Recall that antiderivatives of functions are not unique; they differ from one another by constants. Thus, initial values are very important in determining solutions of differential equations. Slope fields, also called direction fields, are a way to capture the behavior of a whole family of solutions to a particular differential equation with different initial conditions. Euler's Method is a way to approximate solutions to differential equations numerically; it is similar in flavor to Newton's Method.*

- **Reading: Lamar University: Paul Dawkins's Paul's Online Math Notes: Differential Equations: "Basic Concepts: Direction Fields" and "First Order DEs: Euler's Method"** Link: Lamar University: Paul Dawkins's *Paul's Online Math Notes: Differential Equations*: "Basic Concepts: Direction Fields" (HTML) and "First Order DEs: Euler's Method" (HTML)

Instructions: Please click the links above, and read these webpages in their entirety.

Studying these webpages should take approximately 2 hours.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Lecture: YouTube: MIT: Arthur Mattuck's "Lecture 1: The Geometrical View of y'=f(x,y): Direction Fields, Integral Curves" and "Lecture 2: Euler's Method for y'=f(x,y) and Its Generalizations"** Link: YouTube: MIT: Arthur Mattuck's "Lecture 1: The Geometrical View of y'=f(x,y): Direction Fields, Integral Curves" (YouTube) and "Lecture 2: Euler's Method for y'=f(x,y) and Its Generalizations" (YouTube)

Also Available in:
iTunes U

Instructions: Please click on the links above, and watch both lectures. In these videos, Professor Mattuck will explain the concept of direction fields and do several examples. He will state several important principles to keep in mind when sketching slope fields and integral curves and will outline Euler's Method and discuss its error. These are the first and second lectures for a differential equations class.

Viewing these lectures and pausing to take notes should take approximately 2 hours.

Terms of Use: The videos above are released under a Creative Commons Attribution-Share-Alike License 3.0 (HTML). They attributed to MIT and the original versions can be found here (YouTube) and here (YouTube).

- **Interactive Lab: MIT's d'Arbeloff Interactive Math Project: "Euler's Method" Applet** Link: MIT's d'Arbeloff Interactive Math Project: "Euler's Method" Applet (Java)

Instructions: This is an optional resource. Click on the link above to explore Euler's method with this applet. If you need more guidance, click on the "Help" link in the upper right-hand corner of the applet.

You should dedicate approximately 30 minutes to exploring this resource.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

- **Assessment: MIT: Haynes Miller and Arthur Mattuck's Spring 2004 Differential Equations Class: "Problem Set 1"** Link: MIT: Haynes Miller and Professor Arthur Mattuck's Spring 2004 Differential Equations Class: "Problem Set 1" (PDF)

Instructions: Click on the link above, and then find the link to the problem set, marked "Problem Set 1." Do the parts of problems 1 and 2 that are intended for pencil and paper, but ignore references to the "Mathlet." When you are finished, click on the link above again, and find the link to the PDF with solutions to problem set

1.

Completing this assessment should take approximately 15-20 minutes.

Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

6.4 Exponential and Logistic Growth and Applications *A kind of bacteria might have the property where, given sufficient space, every individual produces one additional individual once an hour, so the population of bacteria doubles every hour. If the bacteria are in a petri dish, however, there are space limitations, and the bacteria may reproduce once per hour at first but taper off dramatically as the population grows close to filling the available space. An exponential function describes the former situation, and a logistic function describes the latter situation; such functions may also describe population decreases. The list of possible applications is long, including the decay of radioactive material, the temperature of melting ice or cooling coffee, the number of people who have heard a rumor, and the accrual of interest in a bank account.*

6.4.1 Exponential and Logistic Growth - Web Media: UC College Prep's *Calculus BC II for AP*: "Applications of Integrals: Exponential, Bounded Growth and Decay" and "Applications of Integrals: Logistic Equation and Population Growth" Link: UC College Prep's *Calculus BC II for AP*: "Applications of Integrals: Exponential, Bounded Growth and Decay" (YouTube) and "Applications of Integrals: Logistic Equation and Population Growth" (YouTube)

Instructions: Click on the links above and watch the interactive lectures. You may want to have a pencil and paper close by, as you will be prompted to work on related problems during the lecture.

Studying these resources should take approximately 1 hour.

```
 Terms of Use:  The videos above are licensed under a [Creative
Commons Attribution-NonCommercial-NoDerivatives
License](http://creativecommons.org/licenses/by-nc-nd/3.0/).  They
are attributed to University of California College Prep.
```

- **Reading: Furman University: Dan Sloughter's Difference Equations to Differential Equations: "6.3 Models of Growth and Decay"** Link: Furman University: Dan Sloughter's Difference Equations to Differential Equations: "6.3 Models of Growth and Decay" (PDF)

  Instructions: Click the link above and read this section. This reading covers the topics outlined in sub-subunits 6.4.1-6.4.4.

  Completing this reading should take approximately 30 minutes.

  Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0. It is attributed to Dan Sloughter and the original version can be found here. Please respect the copyright and terms of use displayed on the solutions guide above.

- **Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Matthias Beck's "Differential Equation of Proportional Growth"** Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Matthias Beck's "Differential Equation of Proportional Growth" (HTML)

  Instructions: Click on the link above. Then, click on the "Index" button. Scroll down to "4. Logarithms and Exponentials, Applications," and click button 146 (Differential Equation of Proportional Growth). Do problems 1-4. If at any time a problem set seems too easy for you, feel free to move on.

  Completing this assessment should take approximately 1 hour.

  Terms of Use: Please respect the copyright and terms of use displayed on the webpage above.

6.4.2 Radioactive Decay and Half-Lives: - Lecture: Khan Academy's "Introduction to Exponential Decay" and "Exponential Growth" Khan Academy's "Introduction to Exponential Decay" (YouTube) and "Exponential Growth" (YouTube)

```
 Instructions: Watch these two videos. In the first, you will learn
that the half-life of a substance is the amount of time it takes for
half of the original amount of the substance to decay through
natural processes. (Think of certain radioactive substances which
are very unstable or the carbon measured in carbon-dating.) The
second video goes over an example from biology: exponential growth
of bacteria.

 Watching these lectures and pausing to take notes should take
approximately 30 minutes.

 Terms of Use: The videos above are released under a [Creative
Commons Attribution-NonCommercial-NoDerivs 3.0 Unported
License](http://creativecommons.org/licenses/by-nc-nd/3.0/). They
are attributed to the Khan Academy.
```

6.4.3 Compound Interest - Lecture: Khan Academy's "Compound Interest and e (part 2)" and "Compound Interest and e (part 3)" Khan Academy's "Compound Interest and e (part 2)" (YouTube) and "Compound Interest and e (part 3)" (YouTube)

```
 Instructions: Please click on the links above and  watch Salman
"Compound Interest and e (part 2)" and "Compound Interest and e
(part 3)."  These videos derive the formula for continuously
compounded interest and apply it.

 Watching these video lectures and pausing to take notes should take
approximately 30 minutes.

 Terms of Use: The videos above are released under a [Creative
Commons Attribution-NonCommercial-NoDerivs 3.0 Unported
License](http://creativecommons.org/licenses/by-nc-nd/3.0/).  They
are attributed to the Khan Academy.
```

6.4.4 Epidemiology - Web Media: UC College Prep's Calculus BC II for AP: "Applications of Integrals: Other Examples: Spread of Disease, Rumor" The Saylor Foundation does not yet have materials for this portion of the course. If you are interested in contributing your content to fill this gap or aware of a resource that could be used here, please submit it here.

```
[Submit Materials](/contribute/)
```

- **Assessment: Furman University: Dan Sloughter's Difference Equations to Differential Equations "6.3 Models of Growth and Decay"** Link: Furman University: Dan Sloughter's *Difference Equations to Differential Equations* "6.3 Models of Growth and Decay" (PDF)

  Instructions: This assessment covers subunits 6.4.1-6.4.4. Click the link above and do problems 1 (a, c, e, f), 4, 6-11, 13, and 14. When finished, click here for solutions (courtesy of the author's blog).

  Terms of Use: The article above is released under a Creative Commons Attribution-NonCommercial-ShareAlike License 3.0. It is attributed to Dan Sloughter and the original version can be found here. Please respect the copyright and terms of use displayed on the solutions guide above.

6.4.5 Newton's Law of Cooling - Assessment: Temple University: Gerardo Mendoza's and Dan Reich's Calculus on the Web: Calculus Book II: Matthias Beck and Molly M. Cow's "Population Growth" and Dan Reich's "Newton's Law of Cooling" Link: Temple University: Gerardo Mendoza's and Dan Reich's *Calculus on the Web: Calculus Book II*: Matthias Beck and Molly M. Cow's "Population Growth" (HTML) and Dan Reich's "Newton's Law of Cooling" (HTML)

```
 Instructions: Click on the above link.  Then, click on the "Index"
button.  Scroll down to "4. Logarithms and Exponentials,
Applications," and click button 147 (Population Growth).  Do
problems 1-4.  Next, click button 148 (Newton's Law of Cooling), and
do problems 1-4.  If at any time a problem set seems too easy for
you, feel free to move on.

 Completing these assessments should take approximately 1 hour.
```

- **Assessment: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Growth and Decay"** Link: Clinton Community College: Elizabeth Wood's "Supplemental Notes for Calculus II: Growth and Decay" (PDF)

Also Available in:

HTML

Instructions: Click on the link above, and work through each of the five examples on the page. As in any assessment, solve the problem on your own first. Detailed solutions are given beneath each example.

Completing this assessment should take approximately 30 minutes.

1

When approximating the area under the curve $f(x) = 1 + x^3$ on the interval $0, 4$, using the midpoint rule over four subintervals, one would obtain which of the following?

Choose one answer.

A. 20 ✗

B. $\frac{103}{2}$ ✗

C. 66 ✓

D. $\frac{225}{8}$ ✗

E. None of the above ✗

Question**2**

To numerically approximate solutions to differential equations, it is common to use which of the following?

Choose one answer.

A. Simpson's Rule ✗

B. Rolle's Method ✗

C. Newton's Method ✗

D. Euler's Method ✓

E. None of the above ✗

Question**3**

Find the length of the curve $y = 1 + 6x^{\frac{3}{2}}$ over the interval $0, 1$.

Choose one answer.

A. $\frac{2}{243}(82\sqrt{82} - 1)$ ✓

B. $\frac{4}{243}(82\sqrt{82}-1)$ ✗

C. $\frac{2}{243}(82\sqrt{82}-2)$ ✗

D. $\frac{4}{243}(82\sqrt{82}-2)$ ✗

E. None of the above ✗

Question**4**

What integral could you use to find the length of the curve $x = y + y^3$ over the interval $1 \leq y \leq 4$?

Choose one answer.

A. $\int_1^4 \sqrt{y^4 + 6y^2 + 2}dy$ ✗

B. $\int_1^4 \sqrt{y^4 + 2}dy$ ✗

C. $\int_1^4 \sqrt{9y^4 + 2}dy$ ✗

D. $\int_1^4 \sqrt{9y^4 + 6y^2 + 2}dy$ ✓

E. None of the above ✗

Question**5**

Find the area between the curves $y = x + 1$, $y = 9 - x^2$, $x = -1$, and $x = 2$.

Choose one answer.

A. 18.5 ✗

B. 19.5 ✓

C. 20.5 ✗

D. 21.5 ✗

E. None of the above ✗

Question**6**

Find the area between the curves $y = \sqrt{x}$ and $y = \frac{x}{2}$ from $x = 0$ to $x = 9$.

Choose one answer.

A. $\frac{59}{12}$ ✓

B. $\frac{67}{12}$ ✗

C. 4 ✗

D. $\frac{55}{12}$ ✗

E. None of the above ✗

Question**7**

Find the area between the curves $x = 2y^2$ and $x + y = 1$.

Choose one answer.

	A. $\frac{13}{8}$ ✗
	B. $\frac{11}{8}$ ✗
	C. $\frac{9}{8}$ ✓
	D. $\frac{7}{8}$ ✗
	E. None of the above ✗

Question**8**

Find the average value of the function $f(x) = cos(x)$ on the interval $0, \frac{9\pi}{2}$.

Choose one answer.

	A. $\frac{1}{\pi}$ ✗
	B. $\frac{2}{\pi}$ ✗
	C. $\frac{3}{\pi}$ ✗
	D. $\frac{4}{\pi}$ ✗
	E. None of the above ✓

Question**9**

Find the average value of the function $f(x) = (x-3)^2$ on the interval $2, 5$.

Choose one answer.

	A. 1 ✓
	B. $\frac{3}{2}$ ✗
	C. 2 ✗
	D. 3 ✗
	E. None of the above ✗

Question**10**

If possible, find the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^n}$.

Choose one answer.

	A. $\frac{1}{5}$ ✓
	B. $\frac{3}{5}$ ✗

C. $\frac{-2}{3}$ ✗

D. 3 ✗

E. The series diverges. ✗

Question**11**

If possible, find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n+4^n}{5^n}$.

Choose one answer.

A. $\frac{22}{315}$ ✗

B. $\frac{80}{63}$ ✗

C. $\frac{28}{5}$ ✗

D. $\frac{14}{3}$ ✓

E. The series diverges. ✗

Question**12**

If possible, find the sum of the series $\sum_{n=1}^{\infty} \frac{2}{(n+1)} + \left(\frac{2}{3}\right)^n$.

Choose one answer.

A. 3 ✗

B. $\frac{7}{2}$ ✗

C. 4 ✗

D. $\frac{9}{2}$ ✗

E. The series diverges. ✓

Question**13**

If possible, find the sum of the series $\sum_{n=1}^{\infty} \frac{n(n-1)}{n^3-1}$.

Choose one answer.

A. $\frac{1}{2}$ ✗

B. $\frac{\pi^2}{6}$ ✗

C. 1 ✗

D. 2 ✗

E. The series diverges. ✓

Question**14**

Let $a_n = f(n)$. Which of the following is NOT necessary to use the integral test to check the convergence or divergence of $\sum_{n=1}^{\infty} a_n$? Choose the best answer.

Choose one answer.

A. $f(n)$ is continuous. ✗

B. $f(n)$ is concave up. ✓

C. $f(n)$ is decreasing. ✗

D. $f(n)$ is positive. ✗

E. All of the above ✗

Question**15**

Which of the following best describes the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^4-2}$?

Choose one answer.

A. The sequence converges conditionally. ✓

B. The sequence converges absolutely. ✗

C. The sequence diverges. ✗

D. All of the above ✗

Question**16**

Which of the following best describes the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2n}{5n+2}$?

Choose one answer.

A. The sequence converges conditionally. ✗

B. The sequence converges absolutely. ✗

C. The sequence diverges. ✓

D. All of the above ✗

Question**17**

Which of the following best describes the series $\sum_{n=1}^{\infty} e^{-n^2} n!$?

Choose one answer.

A. The sequence converges conditionally. ✗

B. The sequence converges absolutely. ✓

C. The sequence diverges. ✗

D. All of the above ✗

Question**18**

If the acceleration of a particle with initial velocity 5 is given by $a = 3t^2 + 2t + 1$, how far is it from its initial position at time $t = 3$?

Choose one answer.

A. 41 ✗

B. 42.5 ✗

C. 45 ✗

D. 48.75 ✓

E. None of the above ✗

Question**19**

If the acceleration of a particle with initial velocity 3 is given by $a = 2cos(t) + 1$, then how far is it from its initial position at time $t = 2\pi$?

Choose one answer.

A. $2\pi^2 + 6\pi$ ✓

B. $4\pi^2 + 6\pi$ ✗

C. $2\pi^2 + 3\pi$ ✗

D. $4\pi^2 + 3\pi$ ✗

E. None of the above ✗

Question**20**

Calculate $\int_0^2 e^{2x} dx$.

Choose one answer.

A. 1 ✗

B. $e^4 - 1$ ✗

C. $\frac{e^4}{2} - 1$ ✗

D. $\frac{e^4 - 1}{2}$ ✓

E. None of the above ✗

Question**21**

Calculate $\int_0^5 e^{-3x} dx$.

Choose one answer.

A. $\frac{1}{3}(1 - e^{-15})$ ✓

B. $\frac{-1}{3}(1 - e^{-15})$ ✗

C. $\frac{1}{5}(1-e^{-15})$ ✗

D. $\frac{-1}{5}(1-e^{-15})$ ✗

E. None of the above ✗

Question**22**

Find $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$.

Choose one answer.

A. $e^{\frac{3x}{2}}+C$ ✗

B. $\frac{2}{3}e^{\frac{3x}{2}}+C$ ✗

C. $2e^{\sqrt{x}}+C$ ✓

D. $\frac{e^{\sqrt{x}}}{2}+C$ ✗

E. None of the above ✗

Question**23**

The Fundamental Theorem of Calculus states which of the following?

Choose one answer.

A. If $f(x)$ is differentiable at x_0, then $f'(x)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$. ✗

B. If $f(x)$ is differentiable on (a,b), then there exists c in the interval (a,b) such that $f'(c)=\frac{f(b)-f(}{b-a}$

C. If $f(x)$ has a local max at x_0, then $f'(x_0)=0$ or $f'(x_0)$ is undefined. ✗

D. If $f(x)=F'(x)$, then $\int_a^b f(x)dx=F(b)-F(a)$. ✓

E. None of the above ✗

Question**24**

To find the interval on which $f(x)=\int_0^x \frac{2}{3+4t+5t^2}dt$ is concave upward, an immediate application of which theorem will simplify the problem?

Choose one answer.

A. L'Hopital's Rule ✗

B. Mean Value Theorem ✗

C. Descartes' Rule of Signs ✗

D. Second Fundamental Theorem of Calculus ✓

E. None of the above ✗

Question**25**

The growth rate of a population of bacteria is proportional to the current population. If the initial population was 2000 bacteria and the population after 1 day was 40,000 bacteria, how many bacteria would one expect to find at the end of 2 days?

Choose one answer.

A. 80,000 bacteria ✗

B. 400,000 bacteria ✗

C. 800,000 bacteria ✓

D. 2,000,000 bacteria ✗

E. None of the above ✗

Question**26**

The half-life of Polonium is 102 years. If a 3kg sample now weighs .375 kg, how many years have passed?

Choose one answer.

A. 102 years ✗

B. 204 years ✗

C. 306 years ✓

D. 408 years ✗

E. None of the above ✗

Question**27**

The half-life of Californium is about 900 years. What equation models the amount remaining from a 400g sample after t years?

Choose one answer.

A. $A = 400e^{-\ln(900)t}$ ✗

B. $A = 400e^{-\ln(450)t}$ ✗

C. $A = 400e^{-\ln(\frac{1}{2})t}$ ✗

D. $A = 400e^{-\ln(450t)}$ ✗

E. None of the above ✓

Question**28**

Calculate $\int_0^3 \frac{e^x+e^{-x}}{2}dx$.

Choose one answer.

A. 0 ✗

B. $\frac{e^3-1}{2}$ ✗

C. 1 ✗

D. $\frac{e^3+1}{2}$ ✗

E. None of the above ✓

Question**29**

Calculate $\int_{-1}^{1} \frac{e^x - e^{-x}}{2} dx$.

Choose one answer.

A. 0 ✓

B. $\frac{e^1 - e^{-1}}{2}$ ✗

C. 1 ✗

D. $\frac{e^1 + e^{-1}}{2}$ ✗

E. None of the above ✗

Question**30**

Consider the integral $\int_a^b \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$. Which of the following is a correct characterization of this integral?

Choose one answer.

A. The integral is undefined, if $a < 0$ and $b > 0$. ✗

B. The integral is undefined, if $a < 0$, $b > 0$, and $|a| \neq |b|$. ✗

C. The integral is undefined, only if $a = 0$ or $b = 0$. ✗

D. The integral is undefined, only if a or b is an integer multiple of pi. ✗

E. The integral is defined for all finite values of a and b. ✓

Question**31**

For what values of k is the following integral convergent: $\int_1^{\infty} \frac{1}{x^k} dx$?

Choose one answer.

A. $k < 1$ ✗

B. $k > 1$ ✓

C. $k \geq 1$ ✗

D. $k \leq 1$ ✗

E. None of the above ✗

Question**32**

Evaluate the following integral: $\int_1^{\infty} \frac{\ln x}{x^3} dx$.

Choose one answer.

A. 0 ✗

B. $\frac{1}{4}$ ✓

C. $\frac{1}{2}$ ✗

D. 1 ✗

E. The integral diverges. ✗

Question**33**

Evaluate the following integral: $\int_{-\infty}^{\infty} x^4 e^{-x^5} dx$.

Choose one answer.

A. 0 ✗

B. 1 ✗

C. $\frac{\pi}{2}$ ✗

D. π ✗

E. None of the above ✓

Question**34**

Evaluate the following integral: $\int_{-1}^{4} \frac{1}{x^5} dx$.

Choose one answer.

A. 0 ✗

B. $\frac{1}{5}$ ✗

C. $\frac{1}{4}$ ✗

D. $\frac{15}{32}$ ✗

E. The integral diverges. ✓

Question**35**

Evaluate the following integral: $\int_{0}^{1} \frac{1}{x^{\frac{1}{3}}} dx$.

Choose one answer.

A. 0 ✗

B. $\frac{1}{2}$ ✗

C. 1 ✗

D. $\frac{3}{2}$ ✓

E. The integral diverges. ✗

Question**36**

If $\sum_{n=0}^{\infty} a_n = A$ and $\sum_{n=0}^{\infty} b_n = B$, where both series have only positive terms, which of the following is false?

Choose one answer.

A. $\sum_{n=0}^{\infty} 2a_n = 2A$ ✗

B. $\sum_{n=0}^{\infty} a_n + b_n = A + B$ ✗

C. $\sum_{n=0}^{\infty} a_n - b_n = A - B$ ✗

D. $\sum_{n=0}^{\infty} a_n b_n = AB$ ✓

E. All of the above ✗

Question**37**

Which of the following is a geometric series?

Choose one answer.

A. $\sum_{n=1}^{\infty} \frac{1}{n}$ ✗

B. $\sum_{n=1}^{\infty} \frac{1}{n!}$ ✗

C. $\sum_{n=1}^{\infty} (\frac{1}{k})^n$ ✓

D. $\sum_{n=1}^{\infty} \frac{1}{n^k}$ ✗

E. All of the above ✗

Question**38**

Evaluate $\int (\ln x)^2 dx$.

Choose one answer.

A. $x(\ln x)^2 + 2x + C$ ✗

B. $(\ln x)^2 - 2x \ln x + 2x + C$ ✗

C. $x(\ln x)^2 - 2x \ln x + 2x + C$ ✓

D. $x(\ln x)^2 - 2x \ln x + C$ ✗

E. None of the above ✗

Question**39**

Evaluate $\int x \ln x dx$.

Choose one answer.

A. $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \ln x + C$ ✗

B. $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$ ✓

C. $x^2 \ln x - \frac{1}{4}x^2 \ln x + C$ ✗

D. $\frac{1}{2}x^2 \ln x + C$ ✗

E. None of the above ✗

Question**40**

Evaluate $\int arctan(4t)dt$.

Choose one answer.

A. $\frac{t}{4}arctan(4t) - \frac{1}{8}\ln(1+4t^2) + C$ ✗

B. $\frac{t}{4}arctan(4t) - \frac{1}{8}\ln(1+16t^2) + C$ ✗

C. $tarctan(4t) - \frac{1}{8}\ln(1+4t^2) + C$ ✗

D. $tarctan(4t) - \frac{1}{8}\ln(1+16t^2) + C$ ✓

E. None of the above ✗

Question**41**

Evaluate $\int_0^{3\pi} xsin(3x)dx$.

Choose one answer.

A. π ✓

B. $\frac{3\pi}{2}$ ✗

C. $\frac{4\pi}{3}$ ✗

D. $\frac{5\pi}{2}$ ✗

E. None of the above ✗

Question**42**

Evaluate $\int_0^2 e^{-x}(x^2+1)dx$.

Choose one answer.

A. $3 - 11e^{-2}$ ✓

B. $3 - 13e^{-2}$ ✗

C. $1 - 11e^{-2}$ ✗

D. $1 - 13e^{-2}$ ✗

E. None of the above ✗

Question**43**

If one put $1500 in an account where interest was compounded continuously, how many years would have to pass to raise the value of the account to $4500?

Choose one answer.

A. 1 year ✗

B. 2 years ✗

C. 3 years ✗

D. 4 years ✗

E. It cannot be determined from the information given. ✓

Question**44**

Find the endpoints of the interval of convergence for the following power series: $\sum_{n=1}^{\infty} \frac{2x^n}{n5^n}$.

Choose one answer.

A. $-1, 1$ ✗

B. $-3, 3$ ✗

C. $-5, 5$ ✓

D. $-\infty, \infty$ ✗

E. None of the above ✗

Question**45**

Find the endpoints of the interval of convergence for the following power series: $\sum_{n=1}^{\infty} \frac{(-1)^n x^{3n}}{(3n)!}$.

Choose one answer.

A. $-1, 1$ ✗

B. $-3, 3$ ✗

C. $-5, 5$ ✗

D. $-\infty, \infty$ ✓

E. None of the above ✗

Question**46**

Find the endpoints of the interval of convergence of the following power series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n4^n}(x-4)^n$.

Choose one answer.

A. $-8, 0$ ✗

B. $-4, 4$ ✗

C. $0, 8$ ✓

D. $-\infty, \infty$ ✗

E. None of the above ✗

Question**47**

Solve $\frac{dy}{dx} = \frac{xy}{9 \ln y}$ subject to the initial condition that the solution pass through (0, 1).

Choose one answer.

A. $y = e^{\frac{-x}{9}}$ ✗

B. $y = e^{\frac{-x}{3}}$ ✗

C. $y = e^{\frac{x}{9}}$ ✗

D. $y = e^{\frac{x}{3}}$ ✓

E. None of the above ✗

Question**48**

Solve $y\frac{dy}{dx} = cosx$ subject to the initial condition that the solution pass through (0, 2).

Choose one answer.

A. $y = \sqrt{2sin(x) + 2}$ ✗

B. $y = \sqrt{2sin(x) + 4}$ ✓

C. $y = 2\sqrt{2sin(x) + 1}$ ✗

D. $y = 2cos(x)$ ✗

E. None of the above ✗

Question**49**

Calculate $\int_{\frac{1}{2}}^{\frac{e}{2}} \frac{1}{2x} dx$.

Choose one answer.

A. 1 ✗

B. $\frac{1}{2}$ ✓

C. 2 ✗

D. $\frac{e-1}{2}$ ✗

E. None of the above ✗

Question**50**

Calculate $\int \frac{x}{1+x^2} dx$.

Choose one answer.

A. $\frac{1}{1+x^2} + C$ ✗

B. $\frac{1}{2(1+x^2)} + C$ ✗

C. $ln(1 + x^2) + C$ ✗

D. $\frac{1}{2}ln(1+x^2)+C$ ✓

E. None of the above ✗

Question**51**

Evaluate $\int_e^5 \frac{dx}{x\ln(x)}$.

Choose one answer.

A. $\ln(\ln(5))$ ✓

B. $\frac{\ln(5)}{5} - \frac{1}{e}$ ✗

C. $\ln(5) - 1$ ✗

D. $\ln(\frac{1}{5} - \frac{1}{e})$ ✗

E. None of the above ✗

Question**52**

A bottle of water at 65 degrees Fahrenheit is put in a refrigerator at 35 degrees Fahrenheit. After an hour, the bottle has cooled to 50 degrees Fahrenheit. What will the temperature of the bottle be after another hour?

Choose one answer.

A. 47.5 degrees Fahrenheit ✗

B. 45 degrees Fahrenheit ✗

C. 42.5 degrees ✓

D. 40 degrees Fahrenheit ✗

E. It cannot be determined from the information given. ✗

Question**53**

Find the Cartesian equation for the curve described by the parametric equations $x = 2\sin t$ and $y = \cos^2 t$.

Choose one answer.

A. $\frac{1}{2}x + y = 1$ ✗

B. $2y + x^2 = 2$ ✗

C. $4y + x^2 = 4$ ✓

D. $\arctan\frac{x}{\sqrt{y}} = 1$ ✗

E. None of the above ✗

Question**54**

Find the Cartesian equation for the curve described by the parametric equations $x = \sqrt{t}$ and $y = 3 - t$.

Choose one answer.

A. $y = 3 - \sqrt{x}$ ✗

B. $y = 3 - x$ ✗

C. $x = \sqrt{y+3}$ ✗

D. $x = \sqrt{3-y}$ ✓

E. None of the above ✗

Question**55**

Find the equation of the tangent line to the curve described by $x = t + 1$, $y = t^3 + 1$ at the point corresponding to $t = 2$.

Choose one answer.

A. $y = 12x - 27$ ✓

B. $y = 12x + 27$ ✗

C. $y = 3x - 15$ ✗

D. $y = 3x + 15$ ✗

E. None of the above ✗

Question**56**

Find the equation of the tangent line to the curve described by $x = t^2 + 4$, $y = t^4 - 1$ at the point corresponding to $t = 2$.

Choose one answer.

A. $y = 4x + 1$ ✗

B. $y = 4x - 1$ ✗

C. $y = 2x - 10$ ✗

D. $y = 2x + 10$ ✗

E. None of the above ✓

Question**57**

Consider the curve described by $x = e^t$, $y = \frac{1}{e^t+1}$. At the point corresponding to t = 1, which of the following best describes the curve?

Choose one answer.

A. Concave up and increasing ✗

B. Concave down and increasing ✗

C. Concave up and decreasing ✓

D. Concave down and decreasing ✗

E. None of the above ✗

Question**58**

Consider the curve described by $x = t - 1, y = t^2 - 1$. At the point corresponding to t = 1, which of the following best describes the curve?

Choose one answer.

	A. Concave up and increasing ✓
	B. Concave down and increasing ✗
	C. Concave up and decreasing ✗
	D. Concave down and decreasing ✗
	E. None of the above ✗

Question**59**

Compute $\int \frac{1}{x^2+3x+2} dx$.

Choose one answer.

	A. $\ln \left\|\frac{x+2}{x+1}\right\| + C$ ✗
	B. $\ln \left\|\frac{x+1}{x+2}\right\| + C$ ✓
	C. $\ln \left\|x^2 + 3x + 2\right\| + C$ ✗
	D. $\frac{1}{2x+3} \ln \left\|x^2 + 3x + 2\right\| + C$ ✗
	E. None of the above ✗

Question**60**

Compute $\int \frac{x}{x^2+4x+4} dx$.

Choose one answer.

	A. $\ln \left\|x + 2\right\| + \frac{2}{(x+2)} + C$ ✓
	B. $\ln \left\|x + 2\right\| - \frac{2}{(x+2)} + C$ ✗
	C. $\ln(x^2 + 4x + 4) + C$ ✗
	D. $\frac{1}{2} \ln(x^2 + 4x + 4)$ ✗
	E. None of the above ✗

Question**61**

Compute $\int \frac{x-1}{x(x+1)(x+2)} dx$.

Choose one answer.

	A. $\frac{-3}{2} \ln \left\|x\right\| + 2 \ln \left\|x + 1\right\| - \frac{1}{2} \ln \left\|x + 2\right\| + C$ ✗

B. $\frac{3}{2}\ln|x| - 2\ln|x+1| + \frac{1}{2}\ln|x+2| + C$ ✗

C. $\frac{-1}{2}\ln|x| + 2\ln|x+1| - \frac{3}{2}\ln|x+2| + C$ ✓

D. $\frac{1}{2}\ln|x| - 2\ln|x+1| + \frac{3}{2}\ln|x+2| + C$ ✗

E. None of the above ✗

Question**62**

Find the area enclosed by the curve described by the polar equation $r = sin4\theta$.

Choose one answer.

A. π ✗

B. $\frac{\pi}{2}$ ✓

C. $\frac{\pi}{4}$ ✗

D. $\frac{\pi}{8}$ ✗

E. None of the above ✗

Question**63**

Find the length of the curve described by $r = 3cos\theta$ for $0 \leq \theta \leq \frac{2\pi}{3}$.

Choose one answer.

A. $\frac{\pi}{3}$ ✗

B. $\frac{2\pi}{3}$ ✗

C. π ✗

D. 2π ✓

E. None of the above ✗

Question**64**

Find the Cartesian equation for the curve described by the polar equation $r = sec\theta tan\theta$.

Choose one answer.

A. $x^2 - y^2 = 1$ ✗

B. $x^2 + y^2 = 1$ ✗

C. $x = y^2$ ✗

D. $y = x^2$ ✓

E. None of the above ✗

Question**65**

Find the Cartesian equation for the curve described by the polar equation $r = 4cos\theta + 4sin\theta$.

Choose one answer.

A. $x^2 + y^2 = 4$ ✗

B. $x^2 + y^2 = 8$ ✗

C. $(x-2)^2 + (y-2)^2 = 4$ ✗

D. $(x-2)^2 + (y-2)^2 = 8$ ✓

E. None of the above ✗

Question**66**

Find the Cartesian coordinates of the point described by the polar coordinates: $(-2, \frac{\pi}{3})$.

Choose one answer.

A. $(1, \sqrt{3})$ ✗

B. $(-1, -\sqrt{3})$ ✓

C. $(\sqrt{2}, \sqrt{2})$ ✗

D. $(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$ ✗

E. None of the above ✗

Question**67**

Find the Cartesian coordinates of the point described by the polar coordinates: $(3, \frac{\pi}{4})$.

Choose one answer.

A. $(1, \sqrt{3})$ ✗

B. $(-1, -\sqrt{3})$ ✗

C. $(\sqrt{2}, \sqrt{2})$ ✗

D. $(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$ ✓

E. None of the above ✗

Question**68**

Find the equation of the tangent line to the curve described by $r = cos\theta$, at the point corresponding to $\theta = \frac{\pi}{4}$.

Choose one answer.

A. $y = \frac{1}{2}$ ✓

B. $y = x - 1$ ✗

C. $y = 2x - 1$ ✗

D. $y = 2x + 1$ ✗

E. None of the above ✗

Question**69**

Find the equation of the tangent line to the curve described by $r = 1 - sin\theta$, at the point corresponding to $\theta = \frac{\pi}{6}$.

Choose one answer.

A. $y = x + 1$ ✗

B. $y = x + \frac{1}{2}$ ✗

C. $y = \frac{1}{4}$ ✓

D. $x = \frac{1}{4}$ ✗

E. None of the above ✗

Question**70**

Consider the curve described by $r = 3sin\theta$. At the point corresponding to $\theta = \frac{\pi}{6}$, which of the following best describes the curve?

Choose one answer.

A. Concave up and increasing ✗

B. Concave down and increasing ✓

C. Concave up and decreasing ✗

D. Concave down and decreasing ✗

E. None of the above ✗

Question**71**

Consider the curve described by $r = 2$. At the point corresponding to $\theta = \frac{7\pi}{4}$, which of the following best describes the curve?

Choose one answer.

A. Concave up and increasing ✓

B. Concave down and increasing ✗

C. Concave up and decreasing ✗

D. Concave down and decreasing ✓

E. None of the above ✗

Question**72**

Solve $\frac{dy}{dx} = \frac{-x}{y}$.

Choose one answer.

A. $x^2 + y^2 = C$ ✓

B. $x^2 - y^2 = C$ ✗

C. $x + y^2 = C$ ✗

D. $x^2 + y = C$ ✗

E. None of the above ✗

Question**73**

Solve $\left(\frac{1}{x^2}\right)\frac{dy}{dx} = y$.

Choose one answer.

A. $y = \frac{C \ln x}{x^2}$ ✗

B. $y = \frac{C \ln x}{3x^3}$ ✗

C. $y = Ce^{\frac{x^3}{3}}$ ✓

D. $y = e^{x^2} + C$ ✗

E. None of the above ✗

Question**74**

Which of the following could be a formula for a sequence whose first four terms are 0.7, .91, .973, and .9919?

Choose one answer.

A. $1 - .3n$ ✗

B. $1 - (.3)^n$ ✓

C. $.7 + .21(n - 1)$ ✗

D. $n + .21$ ✗

E. None of the above ✗

Question**75**

What is the seventh term of the sequence given by $a_n = \frac{(-2)^n}{n!}$?

Choose one answer.

A. $\frac{-32}{7}$ ✗

B. $\frac{-64}{343}$ ✗

C. $\frac{-8}{315}$ ✓

D. $\frac{-14}{49}$ ✗

E. None of the above ✗

Question**76**

Find the limit of the sequence given by: $a_n = \frac{cos2n}{1+\sqrt{n}}$.

Choose one answer.

A. 0 ✓

B. 1 ✗

C. π ✗

D. $\frac{1}{2}$ ✗

E. None of the above ✗

Question**77**

Find the limit of the sequence given by $a_n = (1 + \frac{3}{n})^{\frac{1}{n}}$.

Choose one answer.

A. 0 ✗

B. 1 ✓

C. π ✗

D. $\frac{1}{2}$ ✗

E. None of the above ✗

Question**78**

Find the limit of the sequence given by $a_n = \frac{n}{2+\sqrt{n}}$.

Choose one answer.

A. 0 ✗

B. $\frac{1}{2}$ ✗

C. 1 ✗

D. 2 ✗

E. None of the above ✓

Question**79**

Find the limit of the sequence given by $a_n = \frac{\ln n}{\ln n^2}$.

Choose one answer.

A. 0 ✗

B. $\frac{1}{2}$ ✓

C. 1 ✗

D. $\ln 2$ ✗

E. None of the above ✗

Question**80**

What is the volume of the solid generated by rotating the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 2$ about the x-axis?

Choose one answer.

A. $\frac{\pi}{2}$ ✓

B. π ✗

C. $\frac{2\pi}{3}$ ✗

D. $\frac{3\pi}{2}$ ✗

E. None of the above ✗

Question**81**

What integral would you use to find the solid generated by rotating the region bounded by $y = x^4$ and $y = 1$ around the line $y = 2$?

Choose one answer.

A. $\pi \int_0^1 (2 - x^4)^2 dx$ ✗

B. $\pi \int_{-1}^1 (2 - x^4)^2 dx$ ✗

C. $\pi \int_{-1}^1 3 - 4x^4 + x^8 dx$ ✓

D. $\pi \int_{-1}^1 (2 - 1 + x^4)^2 dx$ ✗

E. None of the above ✗

Question**82**

What is the volume of the solid obtained by rotating the region bounded by $x = y^2$ and $x = 1$ around the line $x = 1$?

Choose one answer.

A. $\frac{4\pi}{9}$ ✗

B. $\frac{16\pi}{15}$ ✓

C. $\frac{4\pi}{3}$ ✗

D. $\frac{15\pi}{8}$ ✗

E. None of the above ✗

Question**83**

What integral would you use to find the solid generated by rotating the region bounded by $y = tan^3(x)$, $y = 1$, and $x = 0$ around the line $y = 1$?

Choose one answer.

A. $\pi \int_0^1 (tan^3(x) - 1)^2 dx$ ✗

B. $\pi \int_0^{\frac{\pi}{4}} (tan^3(x) + 1)^2 dx$ ✗

C. $\pi \int_0^1 (1 - tan^3(x))^2 dx$ ✗

D. $\pi \int_0^{\frac{\pi}{4}} (1 - tan^3(x))^2 dx$ ✓

E. None of the above ✗

Question**84**

What is the volume of the solid generated by rotating the region bounded by $y = 4(x-2)^2$ and $y = x^2 - 4x + 7$ around the y-axis?

Choose one answer.

A. 10π ✗

B. 12π ✗

C. 14π ✗

D. 16π ✓

E. None of the above ✗

Question**85**

What integral could be used to calculate the volume of the solid generated by rotating the region bounded by $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$, and $x = 2$ around the line $x = 2$?

Choose one answer.

A. $2\pi \int_0^2 \frac{x-2}{1+x^2} dx$ ✗

B. $\pi \int_0^2 \frac{x-2}{1+x^2} dx$ ✗

C. $2\pi \int_0^2 \frac{2-x}{1+x^2} dx$ ✓

D. $\pi \int_0^2 \frac{2-x}{1+x^2} dx$ ✗

E. None of the above ✗

Question**86**

What integral could you use to calculate the volume of the solid generated by rotating the region bounded by $x = \sqrt{sin(y)}$, $y = 0$, $y = \pi$, $x = 0$ about the line $x = 4$?

Choose one answer.

A. $\pi \int_0^{\pi} 16 - (4 - \sqrt{sin(y)})^2 dy$ ✓

B. $2\pi \int_0^{\pi} (y-4)\sqrt{sin(y)}dy$ ✗

C. $2\pi \int_0^{\pi} (4-y)sin(y)dy$ ✗

D. $\pi \int_0^{\pi} (4-y)\sqrt{sin(y)}dy$ ✗

E. None of the above ✗

Question**87**

What is the volume of the solid generated by rotating the positive region bounded by $x = 4y - y^3$ and $x = 0$ around the x-axis?

Choose one answer.

A. $\frac{64\pi}{15}$ ✗

B. $\frac{128\pi}{15}$ ✓

C. $\frac{64\pi}{3}$ ✗

D. $\frac{128\pi}{3}$ ✗

E. None of the above ✗

Question**88**

Calculate $\int x^2\sqrt{1+x^3}dx.$

Choose one answer.

A. $\frac{3x^2}{\sqrt{1+x^3}} + C$ ✗

B. $\frac{2}{9}(1+x^3)^{\frac{3}{2}} + C$ ✓

C. $\frac{2}{9}x^2(1+x^3)^{\frac{3}{2}} + C$ ✗

D. $\frac{2x}{\sqrt{1+x^3}} + C$ ✗

E. None of the above ✗

Question**89**

Evaluate $\int \frac{(ln(x))^3}{x}dx$.

Choose one answer.

A. $\frac{3}{2}(ln(x))^{\frac{2}{3}} + C$ ✗

B. $\frac{1}{3}(ln(x))^3 + C$ ✗

C. $\frac{1}{4}(ln(x))^4 + C$ ✓

D. $\frac{4}{x^4} + C$ ✗

E. None of the above ✗

Question**90**

Calculate $\int \frac{x}{(4x^2+3)^3} dx$.

Choose one answer.

A. $3\ln(4x^2+3) + C$ ✗

B. $\frac{-1}{16(4x^2+3)^2} + C$ ✓

C. $\frac{-1}{32(4x^2+3)^4} + C$ ✗

D. $\left(\frac{1}{3}\arctan\left(\frac{2}{\sqrt{3}}x\right)\right)^3 + C$ ✗

E. None of the above ✗

Question**91**

Find the indefinite integral: $\int e^x\sqrt{1+e^x}dx$.

Choose one answer.

A. $\frac{3}{2}(1+e^x)^{\frac{3}{2}} + C$ ✗

B. $\frac{1}{2}(1+e^x)^{\frac{3}{2}} + C$ ✗

C. $(1+e^x)^{\frac{1}{2}} + C$ ✗

D. $\frac{2}{3}(1+e^x)^{\frac{3}{2}} + C$ ✓

E. None of the above ✗

Question**92**

What integral could you use to calculate the area of the surface of revolution generated by rotating the curve $y = ln(x), 1 \leq x \leq 3$ around the x-axis?

Choose one answer.

A. $\int_1^3 2\pi x sqrt1 + (\frac{1}{x})^2 dx$ ✗

B. $\int_1^3 \pi ln(x)\sqrt{1+(\frac{1}{x})^2}dx$ ✗

C. $\int_1^3 2\pi ln(x)\sqrt{1+(\frac{1}{x})^2}dx$ ✓

D. $\int_1^3 \pi ln(x)\sqrt{1+(\frac{1}{x})}dx$ ✗

E. None of the above ✗

Question**93**

Find the Taylor series centered at $x = 3$ for the function $f(x) = \ln(2x)$.

Choose one answer.

A. $\ln(6) + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n3^n}(x-3)^n$ ✓

B. $\ln(6) + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}n3^{n+1}(x-3)^n$ ✗

C. $\ln(6) + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n(n+1)}(x-3)^n$ ✗

D. $\ln(6) + \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n(n+1)}(x-3)^n$ ✗

E. None of the above ✗

Question**94**

Find the Taylor series centered at $x = \frac{\pi}{2}$ for $f(x) = cos2x$.

Choose one answer.

A. $\sum_{n=0}^{\infty} \frac{(-2)^{2n+1}}{(2n+1)!}(x-\frac{\pi}{2})^{2n+1}$ ✗

B. $\sum_{n=0}^{\infty} \frac{(2)^{2n+1}}{(2n+1)!}(x-\frac{\pi}{2})^{2n+1}$ ✗

C. $\sum_{n=0}^{\infty} \frac{(-2)^{2n}}{(2n)!}(x-\frac{\pi}{2})^{2n}$ ✓

D. $\sum_{n=0}^{\infty} \frac{(2)^{2n}}{(2n)!}(x-\frac{\pi}{2})^{2n}$ ✗

E. None of the above ✗

Question**95**

Evaluate $\int sin^3(x)cos^2(x)dx$.

Choose one answer.

A. $\frac{1}{9}sin^5(x) - \frac{2}{7}sin^7(x) + C$ ✗

B. $\frac{1}{5}sin^5(x) - \frac{1}{3}sin^3(x) + C$ ✗

C. $\frac{1}{5}cos^5(x) - \frac{1}{3}cos^3(x) + C$ ✓

D. $\frac{1}{9}cos^5(x) - \frac{2}{7}cos^7(x) + C$ ✗

E. None of the above ✗

Question**96**

Calculate $\int_0^{\frac{\pi}{2}} cos^2(t)dt$.

Choose one answer.

A. $\frac{\pi}{8}$ ✗

B. $\frac{\pi}{4}$ ✓

C. $\frac{\pi}{3}$ ✗

D. $\frac{\pi}{2}$ ✗

E. None of the above ✗

Question**97**

Calculate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} cot^2(x)dx$.

Choose one answer.

A. $\frac{\pi}{6} - \sqrt{6}$ ✗

B. $\sqrt{6} - \frac{\pi}{6}$ ✗

C. $\frac{\pi}{3} - \sqrt{3}$ ✗

D. $\sqrt{3} - \frac{\pi}{3}$ ✓

E. None of the above ✗

Question**98**

Evaluate .

Choose one answer.

A. $\frac{1}{3}(x^2 - 9)\sqrt{x^2 + 9} + C$ ✗

B. $\frac{1}{3}(x^2 - 18)\sqrt{x^2 + 9} + C$ ✓

C. $\frac{3}{2}x^2 + 9)^{\frac{3}{2}} + C$ ✗

D. $\frac{1}{3}(x^2 + 6x + 9)\sqrt{x^2 + 9} + C$ ✗

E. None of the above ✗

Question**99**

Evaluate $\int_0^{\frac{2}{3}} x^3\sqrt{4-9x^2}dx$.

Choose one answer.

A. $\frac{63}{1216}$ ✗

B. $\frac{61}{1215}$ ✗

C. $\frac{65}{1216}$ ✗

D. $\frac{64}{1215}$ ✓

E. None of the above ✗

Question**100**

Evaluate $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx$.

Choose one answer.

A. π ✗

B. $\frac{\pi}{2}$ ✗

C. 1 ✗

D. $\frac{\pi}{3}$ ✓

E. None of the above ✗

Question**101**

Recalling that the force exerted by a spring as it is stretched is $F(x) = -kx$, where k is the spring constant and x is the displacement from equilibrium, what integral would you use to find the work done in stretching a spring with constant k 3 units from equilibrium?

Choose one answer.

A. $\int_0^3 -kdx$ ✗

B. $\int_0^3 kxdx$ ✗

C. $\int_0^3 -kxdx$ ✓

D. $\int_0^3 -kx^2dx$ ✗

E. None of the above ✗